Department of Economics

Essays on Estimating Production Functions

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Aalto University

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Abstract

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Abstract

In the first essay I estimate production functions of multiproduct firms when technologies are product-specific but inputs are observable only at the firm-level. I provide an estimation strategy that solves for the unobservable inputs while correcting for the well-known simultaneity, collinearity and omitted price problems in production function estimation. The key insights of the estimation strategy are, first, using output demand estimates in identifying the product-level input allocations and production functions, and second, using an inverse of the production function to control for endogeneity.

The second essay describes the biases that arise when production functions are estimated under the standard assumption of a firm-level technology, while the true technologies are product-specific. The assumption of a firm-level technology implies that the technology parameters are identical across the various goods produced in the industry, and that a multiproduct firm produces all of its output with a single technology. To examine the implications of these simplifying assumptions, I estimate a firm-level production function on a dataset generated of an industry where two types of goods are produced with product-specific Cobb-Douglas production functions. I find that the biases in the estimated firm-level parameters are substantial even when the true product-specific technologies are very similar. The directions and the magnitudes of the biases are determined by intricate functions of the true product-specific technologies and the product scopes of the firms in the industry. The estimated productivity levels have a relatively low correlation with the true firm-level productivity levels when the firms' product scopes are heterogeneous, as they usually are.

The third essay estimates the range of productivity gains achieved by information technology investments in the Finnish manufacturing sector. The contribution is to provide estimates of IT's productivity effects while accounting for some of the key characteristics of IT, i.e., that returns to IT depend on previous IT or complementary investments, come with lags, and, due to the aforementioned factors, are heterogeneous across firms and over time. I find that the productivity effects of IT range from negative to positive. For example, most firms obtain a negative productivity effect in the first year after the investment, which may be due to disruption in the production process caused by the implementation of the IT investment. Two years after the IT investment was made, most firms attain a positive productivity effect. In the third year after the investment, almost all firms gain a positive productivity effect. The estimation results suggest that the common practice of estimating a single output elasticity for an IT stock that is constructed as a linear function of the IT investments is unlikely to provide a truthful description of the productivity effects of IT.

Keywords Production function, productivity, multiproduct firm, information technology

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I spent the fifth year of my graduate studies visiting the Economics Department at the University of Michigan in Ann Arbor, where Prof. Daniel Ackerberg and Prof. Jeremy Fox kindly hosted my stay. I'm grateful for the instructive discussions I had with them. I thank Dan also for pre-examining this thesis.

Prof. Frédèric Warzynski was the other thesis pre-examiner, and agreed to be also my opponent in the thesis defense. I'm very happy that he took up this task, especially on such a swift pace.

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My spouse Pete has been a constant supporter and motivator through my studies. I've been fortunate to get his help with practical problems, such as fixing bugs in the Matlab code – I'm sorry Pete for those beautiful summer evenings spoilt at the office. But, more importantly, I appreciate the example Pete has given me with his humane worldview and intelligent attitude towards external shocks in life. Thank you Pete for always being there for me.

Helsinki, October 2014

Nelli Valmari

List of essays

This dissertation consists of an introduction and the following three essays:

Essay 1: "Estimating Production Functions of Multiproduct Firms", unpublished.

Essay 2: "Misspecified Production Functions: Product- vs. Firm-level Technologies", unpublished.

Essay 3: "Heterogeneous Productivity of Information Technology", unpublished.

1 Introduction

Production technologies, described by production functions, are a key determinant of economic efficiency and growth in firms, industries, and economies. When resources are limited, the quantity of output can be increased either by implementing more efficient production technologies, or by allocating the limited resources more efficiently among the producers.

A production function relates inputs, such as labor and capital, to the produced output. The production function defines the rates at which the inputs augment the output, i.e., the output elasticities of the inputs. Any output, or deficiency of output, that the inputs and their output elasticities do not account for is captured by the producer's total factor productivity or, in short, productivity.

Production functions are an important tool to understand how output is determined by various supply-side factors and production technologies. Production function estimates characterize how various inputs, such as employees with different levels of education, or new technologies, such as process innovations or novel management practices, affect firms' output. The sum of the output elasticities determine whether the production technology is subject to economies of scale. Total factor productivity is captured by the residual of the estimated production function, which describes productivity differences between producers. The production function estimates, together with information on input costs, can be used to compute marginal costs of production.

Economists estimate firms' production functions to evaluate how changes in the economic environment, new policies, or various production related factors, such as process innovations, affect firms' and markets' performance. An example of a policy that changes the economic environment is liberalization of international trade, which has been found to lead to increased competition between firms and, as a result, higher productivity (Bernard, Redding and Schott, 2011, and Mayer, Melitz and Ottaviano, 2013). Competition enhances firms' productivity and reduces productivity differences between firms also within national markets (Berger and Hannan, 1998, Dunne, Klimek and Schmitz, 2010, Schmitz, 2005, and Syverson, 2004). Competition can have a causal effect on firms' productivity and, for example, shift the productivity distribution to the right. Alternatively, it can drive the least productive firms out of the industry, which truncates the productivity distribution from the left tail.

Firms make various investments and changes in their production processes to improve productivity. New management practices and investments in organizational capital, research and development in new production processes and products, and adoption of information technology are typical examples of productivity-enhancing investments. They have been found to be complementary to each other, which means that the investments' productivity effects are greater when implemented together (Bloom, Sadun and Van Reenen, 2012, Bresnahan, Brynjolfsson and Hitt, 2002). Productivity effects of various investments are of interest to policy-makers who design innovation policies such as R&D subsidies.

An interesting finding made in many productivity studies is that even within narrowly defined industries, productivity differentials between firms are substantial and persistent (Doms and Bartelsman, 2000; Syverson, 2011). Syverson (2004) finds that in four-digit SIC industries of the US manufacturing sector, on average, the plant at the 90th percentile of the productivity distribution produces almost twich as much as the plant at the 10th percentile with the same measured inputs. This finding suggests that resource allocation within industries is not efficient. So far the productivity literature has not been able to show how such productivity differences arise, and why they persist.

1.1 Estimation challenges

The current literature recognizes several identification issues that challenge the estimation of production functions. Marschak and Andrews (1944) first pointed out that inputs are not independent variables because firms set them with the aim of maximizing profit. More precisely, inputs are endogeneous to the productivity level that is unobservable to the econometrician. This endogeneity bias, often referred to as the simultaneity or transmission bias, is the identification problem most carefully considered in the production function literature. Traditional solutions are using instrumental variables or estimating a fixed effects production function model (Mundlak, 1961). In practice, however, these solutions have not performed well. Data sets usually fall short of appropriate instruments for the endogenous variables. Furthermore, the fixed effects model relies on an unrealistic assumption of firm productivity being constant over time. Failure to correct for the simultaneity bias leads to overestimated production function parameters for the flexible inputs such as materials and possibly also labor.

Another endogeneity problem is the selection bias. As first discussed by Wedervang (1965), econometricians do not observe a random sample of firms. A firm's decision to be active in the market depends on its productivity level as well as its fixed input stocks. Firms with a large capital stock may find it profitable to stay active in the market even if they face a negative productivity shock, while the same holds for firms with a small capital stock that face a positive productivity shock. Hence the fixed input stocks and the unobservable productivity levels of the firms observed are negatively correlated. If firm selection is not accounted for, the production function parameters for the fixed inputs, such as capital, are overestimated.

Olley and Pakes (1996, henceforth OP) were the first to correct for the selection bias, while also controlling for the simultaneity of inputs with a novel structural method. To take account of selection OP estimate survival probabilities for the observed firms. The insight that allows them to correct the simultaneity problem is that a firm chooses its investment level as a function of its productivity. Hence the firm's demand for investment, which OP write as a nonparametric function, can be used to back out the unobservable productivity. The key assumptions that enable this identification strategy are (1) strict monotonicity of investment in productivity, (2) productivity as the only unobservable in investment demand, and (3) the timing of investment (labor) choices before (after) the productivity shock. To relax the rather strict assumption of a monotonic investment function, Levinsohn and Petrin (2003, henceforth LP) propose using demand for intermediate inputs, rather than investment, in inverting out productivity. Wooldridge (2009) shows how the two-step estimators of OP and LP can be implemented in one step to improve efficiency.

Another type of identification problem is the omitted price bias, which occurs whenever the production function is estimated using sales revenue and/or input expenditure data, and output and/or input prices are not equal across firms. Harrison (1994) discusses the bias with input prices, and Klette and Griliches (1996) with output prices. Despite the considerable biases these inter-firm price differentials can induce, they have been ignored to a large extent in the empirical literature. The explanation is again largely practical: output and input are often measured in sales revenue and expenditures only.

The most recently discussed identification problem concerns firms' endogeneous product selection. Bernard, Redding and Schott (2009) note that most firms make production decisions at a more disaggregated level than what is observed in the data and therefore studied in the productivity literature. They consider single-product firms that choose one out of two heterogeneous goods based on the productivity of the firm, as well as the production technologies and demand for the goods. Bernard, Redding and Schott derive the productivity bias that arises in revenue production function estimation when endogeneous product selection is not accounted for. The so-called product bias is determined, not surprisingly, by the same factors that influence product selection. The empirical implications of ignoring product endogeneity have not been considered.

1.2 Overview of the essays

This thesis consists of three essays on production function estimation. Two of the essays consider how the existence of product-specific production technologies and multiproduct firms can be taken into account in production function estimation, and what the implications are if they are ignored. The third essay evaluates the range of returns to information technology investments by means of production function estimation.

In the first essay I estimate production functions of multiproduct firms when technologies are product-specific but inputs are observable only at the firm-level. I provide an estimation strategy that solves for the unobservable inputs while correcting for the well-known simultaneity, collinearity and omitted price problems in production function estimation. The key insights of the estimation strategy are, first, using output demand estimates in identifying the product-level input allocations and production functions, and second, using an inverse of the production function to control for endogeneity. Multiproduct firms constitute a considerable share of firms, and even a greater share of production. Estimates of production functions and the implied productivity distributions serve as input for numerous economic studies.

The second essay describes the biases that arise when production functions are estimated under the standard assumption of a firm-level technology, while the true technologies are product-specific. The assumption of a firm-level technology implies that the technology parameters are identical across the various goods produced in the industry, and that a multiproduct firm produces all of its output with a single technology. To examine the implications of these simplifying assumptions, I estimate a firm-level production function on a dataset generated of an industry where two types of goods are produced with product-specific Cobb-Douglas production functions. I find that the biases in the estimated firm-level parameters are substantial even when the true product-specific technologies are very similar. The directions and the magnitudes of the biases are determined by intricate functions of the true product-specific technologies and the product scopes of the firms in the industry. The estimated productivity levels have a relatively low correlation with the true firm-level productivity levels when the firms' product scopes are heterogenous, as they usually are.

The third essay estimates the range of productivity gains achieved by information technology investments in the Finnish manufacturing sector. The contribution is to provide estimates of IT's productivity effects while accounting for some of the key characteristics of IT, i.e., that returns to IT depend on previous IT or complementary investments, come with lags, and, due to the aforementioned factors, are heterogenous across firms and over time. I find that the productivity effects of IT range from negative to positive. For example, most firms obtain a negative productivity effect in the first year after the investment, which may be due to disruption in the production process caused by the implementation of the IT investment. Two years after the IT investment was made, most firms attain a positive productivity effect. In the third year after the investment, almost all firms gain a positive productivity effect. The estimation results suggest that the common practice of estimating a single output elasticity for an IT stock that is constructed as a linear function of the IT investments is unlikely to provide a truthful description of the productivity effects of IT.

References

Berger, A. N., and T. H. Hannan (1998): "The Efficiency Cost of Market Power In The Banking Industry: A Test Of The "Quiet Life" And Related Hypotheses," *Review of Economics and Statistics*, 80(3), 454–465.

Bernard, A. B., S. J. Redding, and P. K. Schott (2009): "Products and Productivity," *Scandinavian Journal of Economics*, 111(4), 681–709.

Bloom, N., R. Sadun, and J. V. Reenen (2012): "Americans Do IT Better: US Multinationals and the Productivity Miracle," *American Economic Review*, 102(1), 167–201.

Bresnahan, T. F., E. Brynjolfsson, and L. M. Hitt (2002): "Information Technology, Workplace Organization, And The Demand For Skilled Labor: Firm-Level Evidence," *Quarterly Journal of Economics*, 117(1), 339–376.

Doms, M., and E. J. Bartelsman (2000): "Understanding Productivity: Lessons from Longitudinal Microdata," *Journal of Economic Literature*, 38(3), 569–594.

Dunne, T., S. Klimek, and J. James Schmitz (2010): "Competition and Productivity: Evidence from the Post WWII U.S. Cement Industry," Working Paper 10-29, Center for Economic Studies, U.S. Census Bureau.

Harrison, A. E. (1994): "Productivity, imperfect competition and trade reform: Theory and evidence," *Journal of International Economics*, 36(1-2), 53–73.

Klette, T. J., and Z. Griliches (1996): "The Inconsistency of Common Scale Estimators When Output Prices Are Unobserved and Endogenous," *Journal of Applied Econometrics*, 11(4), 343–361.

Levinsohn, J., and A. Petrin (2003): "Estimating Production Functions Using Inputs to Control for Unobservables," *Review of Economic Studies*, 70(2), 317–341.

Marschak, J., and J. William H. Andrews (1944): "Random Simultaneous Equations and the Theory of Production," *Econometrica*, 12(3-4), 143–205.

Mayer, T., M. Melitz, and G. I. P. Ottaviano (2014): "Market Size, Competition, and the Product Mix of Exporters," *American Economic Review*, 104(2), 495–536.

Mundlak, Y. (1961): "Empirical Production Function Free of Management Bias," *Journal of Farm Economics*, 43(1), 44–56.

Olley, G. S., and A. Pakes (1996): "The Dynamics of Productivity in the Telecommunications Equipment Industry," *Econometrica*, 64(6), 1263–1297.

Schmitz, James A., J. (2005): "What Determines Productivity? Lessons from the Dramatic Recovery of the U.S. and Canadian Iron Ore Industries Following Their Early 1980s Crisis," *Journal of Political Economy*, 113(3), 582–625.

Syverson, C. (2004a): "Market Structure and Productivity: A Concrete Example," *Journal of Political Economy*, 112(6), 1181–1222.

——— (2004b): "Product Substitutability and Productivity Dispersion," *Review of Economics and Statistics*, 86(2), 534–550.

——— (2011): "What Determines Productivity?," *Journal of Economic Literature*, 49(2), 326–365.

WEDERVANG, F. (1965): *Development of a population of industrial firms: the structure of manufacturing industries in Norway, 1930-1948.*, Scandinavian university books. Universitetsforlaget.

WOOLDRIDGE, J. (2009): "On estimating firm-level production functions using proxy variables to control for unobservables," *Economics Letters*, 104(3), 112–114.

Essay 1

Estimating Production Functions of Multiproduct Firms

Unpublished

Estimating Production Functions of Multiproduct Firms*

Nelli Valmari†

July 17, 2014

Abstract

Multiproduct firms constitute a considerable share of firms, and even a greater share of production. Estimates of production functions and the implied productivity distributions serve as input for numerous economic studies. I estimate production functions of multiproduct firms when technologies are product-specific but inputs are observable only at the firm-level. I provide an estimation strategy that solves for the unobservable inputs while correcting for the well-known simultaneity, collinearity and omitted price problems in production function estimation. The key insights of the estimation strategy are, first, using output demand estimates in identifying the product-level input allocations and production functions, and second, using an inverse of the production function to control for endogeneity.

Keywords: Multiproduct firm, production function, productivity

JEL codes: D24, L11, L25

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1 Introduction

A substantial share of firms is multiproduct firms,[1] and even a greater share of goods is provided by these multiproduct producers. For example, in the US manufacturing sector in 1987 to 1997, 39% of the firms manufactured more than one product title, while these multiproduct firms accounted for 87% of the sector's output (Bernard, Redding and Schott, 2010). In a large sample of Finnish manufacturing plants on years 2004 to 2011,[2] more than 60% of the plants produce at least two product titles. The product scopes range up to 82 titles, and the average product scope of multiproduct firms is 4.3 titles. In international trade multiproduct firms are even more widely present: they accounted for more than 99% of the US exports in 2000 (Bernard, Jensen, Redding and Schott, 2007). Moreover, the product assortments and their output shares vary both across firms,[3] and across time (Bernard, Redding and Scott, 2010).

Despite the empirical fact that multiproduct firms are prevalent, and hence many firms are likely to use several production technologies,[4] the standard practice in production function estimation is to assume that all firms are single-product firms with a single production technology. Most often the output variable is the sum of sales revenue from the various products, and hence the production functions are estimated at the firm-level. The reason for this is pragmatic: to the best of my knowledge, there is no dataset that reports input allocation at the product-firm level for a cross-section of firms.

Unfortunately, the standard practice of ignoring product-level production technologies, and assuming firm-level production functions instead, is likely to have severe implications on production function estimates. Using simulations, Valmari (2014) finds that the biases in the estimated firm-level parameters are substantial even when the true product-level technologies are very similar. The directions and the magnitudes of the biases are determined by intricate functions of the true product-level technologies and the product scopes of the firms in the industry. The estimated productivity levels have a relatively low correlation with the true firm-level productivity levels when the firms' product scopes are

[1] Multiproduct firms exist due to economies of scope. See, for example, Panzar, 1989 for how production technology affects firm and industry structure.

[2] For the description of this data used in this paper, see section 5.1

[3] This is an observation on the data used in this paper.

[4] Hence I may also adopt the term *multitechnology firm* in this paper, but as *multiproduct firm* is already an established term in the literature, and also refers to the fact that these firms sell their goods in various product markets, I stick to the term *multiproduct firm.*

heterogeneous, as they usually are.

In this paper I estimate product-level production functions of firms that are mostly multiproduct producers. I provide a simple structural estimation strategy for product-level production functions when factors of production are observed only at the firm- or establishment-level, which is typical of most micro-level datasets. The challenges consist of solving for the unobservable product-level inputs and, as always in production function estimation, controlling for endogeneity problems, i.e., the endogeneity of inputs to the unobservable productivity. The first key insight underlying my estimation strategy is that by inverting the production function, the very definition of productivity can be used to control for the unobservable productivity level. The second insight that is that, once one can control for the unobservable productivity level, the demand for the final goods can be used to identify the unobservable input allocation as well as the production functions.

I demonstrate the method by using Finnish manufacturing data with output quantities and prices observed at the product-plant-level, and input quantities and prices at the plant-level. I estimate the product-level production functions used in two industries: "Sawmilling and planing of wood" (PRODCOM 161) and "Manufacture of products of wood, cork, straw and plaiting materials" (PRODCOM 162). The empirical findings suggest that production functions should be estimated at the product- instead of the firm-level, and that multiproduct firms use multiple production technologies.

Production function estimates and the implied productivity distributions serve as input for various economic studies. Effects of a new technology or how a change in the level competition affects firms' productivity, market outcomes, and total welfare are typical examples. Productivity distributions speak to the question of how efficiently resources are allocated within industries. One stylized fact of the production function literature is that even within narrowly defined industries, productivity differentials between firms are substantial and persistent (Doms and Bartelsman, 2000; Syverson, 2011). Syverson (2004) finds that in four-digit SIC industries of the US manufacturing sector, on average, the plant at the 90th percentile of the productivity distribution produces almost twich as much as the plant at the 10th percentile with the same measured inputs. Hsieh and Klenow (2009) report even higher productivity differentials for China and India where, on average, the plant at the 90th percentile is more than five times as productive as the plant at the 10th percentile. Another stylized fact is that competition within the industry is correlated with productivity, and that competition narrows the productivity distribution.[5] Competition can have a causal effect on firms' productivity, i.e., shift the productivity distribution to the right. Alternatively, it can drive the least productive firms out of the industry, which truncates the productivity distribution from the left tail. Nevertheless, lack of competition has not been identified as the cause of the wide productivity distributions reported. This makes the first stylized fact of wide productivity distributions even more surprising. My estimation strategy may be used to examine whether some of the surprisingly large productivity differentials may be an outcome of incorrectly assuming industry- instead of product-specific production function parameters.

Accounting for product-specificity in production enables economists to study also new economic questions. For example, we don't yet fully understand what economic factors determine firms' productivity evolution and the productivity differentials observed between firms. As many key strategic decisions are made at the product-level, understanding production and profit maximization at the product-level is essential. Due to the practice of estimating productivity at the firm-level, the product-level factors are still largely unexplored. Furthermore, endogeneous product choices by firms, and how these endogeneities can be taken into account in, for example, demand estimation, entry models and policy simulations, have become a subject of interest in the recent industrial organization literature.[6] So far, however, the role of product-specific technology on product choice has not been studied.

In the next section I review shortly the literature on identification of production functions and production by multiproduct firms. The model and the estimation strategy are presented in sections 3 and 4. In section 5, I introduce the dataset and provide further details of the estimation procedure. Empirical results are presented in section 6. Section 7 provides a discussion on how the identifying assumptions of my estimation strategy relate to the current production function literature, and particularly how they compare with the identifying assumptions underlying the empirical model of multiproduct firms of De Loecker, Goldberg, Khandelwal and Pavcnik (2012). Section 8 concludes.

[5] See Berger and Hannan, 1998; Dunne, Klimek and Schmitz, 2010; Schmitz, 2005; and Syverson, 2004.
[6] See Ackerberg, Crawford and Hahn, 2011; Draganska, Mazzeo and Seim, 2009; and Seim, 2006.

2 Literature

This paper relates to two bodies of literature. The first is about identification and estimation of production functions. The second is about production by multiproduct firms.

2.1 Identification of production functions

The current literature recognizes several identification issues that challenge the estimation of production functions. Marschak and Andrews (1944) first pointed out that inputs are not independent variables because firms set them with the aim of maximizing profit. More precisely, inputs are endogeneous to the productivity level that is unobservable to the econometrician. This endogeneity bias, often referred to as the simultaneity or transmission bias, is the identification problem most carefully considered in the production function literature. Traditional solutions are using instrumental variables or estimating a fixed effects production function model (Mundlak, 1961). In practice, however, these solutions have not performed well. Data sets usually fall short of appropriate instruments for the endogenous variables. Furthermore, the fixed effects model relies on an unrealistic assumption of firm productivity being constant over time. Failure to correct for the simultaneity bias leads to overestimated production function parameters for the flexible inputs such as materials and possibly also labor.

Another endogeneity problem is the selection bias. As first discussed by Wedervang (1965), econometricians do not observe a random sample of firms. A firm's decision to be active in the market depends on its productivity level as well as its fixed input stocks. Firms with a large capital stock may find it profitable to stay active in the market even if they face a negative productivity shock, while the same holds for firms with a small capital stock that face a positive productivity shock. Hence the fixed input stocks and the unobservable productivity levels of the firms observed are negatively correlated. If firm selection is not accounted for, the production function parameters for the fixed inputs, such as capital, are overestimated.

Olley and Pakes (1996, henceforth OP) were the first to correct for the selection bias, while also controlling for the simultaneity of inputs with a novel structural method. To take account of selection OP estimate survival probabilities for the observed firms. The insight that allows them to correct the simultaneity problem is that a firm chooses its investment level as a function of its productivity. Hence the firm's demand for investment, which OP write as a nonparametric function, can be used to back out the unobservable productivity. The key assumptions that enable this identification strategy are (1) strict monotonicity of investment in productivity, (2) productivity as the only unobservable in investment demand, and (3) the timing of investment (labor) choices before (after) the productivity shock. To relax the rather strict assumption of a monotonic investment function, Levinsohn and Petrin (2003, henceforth LP) propose using demand for intermediate inputs, rather than investment, in inverting out productivity. Wooldridge (2009) shows how the two-step estimators of OP and LP can be implemented in one step to improve efficiency.

Ackerberg, Caves and Frazer (2006, henceforth ACF) observe that the identification strategies of OP, and especially of LP, suffer from collinearity problems. ACF point out that in both estimation strategies the static labor input is collinear with the nonparametric input demand function that is inverted for the unobservable productivity. ACF provide an alternative identification strategy that uses the insights of OP and LP but with slightly modified timing assumptions avoids the aforementioned collinearity problem. However, they also acknowledge that if a gross output production function with more than one flexible input is estimated, there is one identification problem remaining. As shown by Bond and Söderbom (2005), in the absence of inter-firm variation in the input prices, flexible inputs are collinear with each other and with any fixed inputs.

Some studies attempt to control for the collinearity problem by estimating a value added production function that has only one flexible input. However, Gandhi, Navarro and Rivers (2013) show that the value added specification is not a resolution to the collinearity problem, but induces a so-called value added bias instead. In excluding flexible inputs, which are collinear with productivity and other inputs, the degree of productivity heterogeneity is overstated and the elasticity estimates for the fixed inputs are biased. Gandhi, Navarro and Rivers show that if the value added bias is not corrected, the estimated inter-firm productivity differences are orders of magnitude larger, and even of opposite sign, than the productivity differences obtained when correcting for the bias. They provide a strategy to correct for the collinearity and simultaneity problems for both gross output and value added specifications. Gandhi, Navarro and Rivers make the same assumptions regarding timing of input choices and evolution of productivity as ACF, but identification is based on a transformation of the firm's short-run first order conditions.

Also the so-called monotonicity assumption of the aforementioned proxy estimators has been contested. Ornaghi and Van Beveren (2011) compare the performance of the proxy method proposed by OP, and modifications to it by LP, ACF, and Wooldridge. The methods differ in the proxy variables, assumptions on the timing of input decisions and when investments translate into productive capital, and moment conditions. However all the estimators are based on the so-called monotonicity assumption that the proxy variable monotonically increases in the unobservable productivity term. As noted by Ornaghi and Van Beveren, if the monotonicity assumption is violated, the estimators yield inconsistent estimates. They propose a diagnostic tool for testing whether the monotonicity assumption holds for the estimators. Ornaghi and Van Beveren find that the assumption fails to hold in the majority of cases they consider. The assumption holds in all three industries examined in at least 90% of the cases only for three estimators: OP/LP with non-linear least squares, OP/LP with GMM, and Wooldridge's one-step estimator with the assumptions of OP. Furthermore, there is a large degree of heterogeneity in the results, which indicates that the timing assumptions and the choice of the estimator affect the estimates.

Another type of identification problem is the omitted price bias, which occurs whenever the production function is estimated using sales revenue and/or input expenditure data, and output and/or input prices are not equal across firms. Harrison (1994) discusses the bias with input prices, and Klette and Griliches (1996) with output prices. Despite the considerable biases these inter-firm price differentials can induce, they have been ignored to a large extent in the empirical literature. The explanation is again largely practical: output and input are often measured in sales revenue and expenditures only.

The most recently discussed identification problem concerns firms' endogeneous product selection. Bernard, Redding and Schott (2009) note that most firms make production decisions at a more disaggregated level than what is observed in the data and therefore studied in the productivity literature. They consider single-product firms that choose one out of two heterogeneous goods based on the productivity of the firm, as well as the production technologies and demand for the goods. Bernard, Redding and Schott derive the productivity bias that arises in revenue production function estimation when endogeneous product selection is not accounted for. The so-called product bias is determined, not surprisingly, by the same factors that influence product selection. The empirical implications of ignoring product endogeneity have not been considered.

Also the functional form assumptions have been challenged. When estimating the Cobb-Douglas production function the vast majority of firm-level studies assume that productivity is Hicks neutral, i.e., that a change in productivity does not change the input shares used. Using data on U.S. manufacturing plants Raval (2012) shows that a CES production function with labor augmenting productivity differences better accounts for the characteristics of the firms observed, as compared to the Hicks neutral Cobb-Douglas technology.

2.2 Multiproduct firms

A large share of the recent literature on multiproduct firms is written in the context of international trade, perhaps because international trade flows are dominated by multi-product firms. In 2000, firms that exported more than one product title, as defined at the ten-digit level, accounted for more than 99% of the US export value (Bernard, Jensen, Redding and Schott, 2007). A number of studies centers on how reductions in barriers to international trade affect firms' productivity and product scope. Nearly every study finds that as reductions in trade barriers lead to increased competition, the firms that remain active become more productive. Theoretical findings on the product scope, which is a potential channel for productivity effects to take place, are mixed. As a consequence to reductions in trade barriers, product scopes are found to decrease,[7] increase,[8] or both.[9] Empirical evidence indicates that increased competition drives firms to concentrate on the goods they are most competent in and drop the least productive products from the selection of exported goods,[10] unless industrial regulations hinders firms from doing so (Goldberg, Khandelwal, Pavcnik and Topalova, 2010). In other words, empirical evidence suggests that firms' productivity across goods vary.

Multiproduct firms are widely present also within national markets. As in the global markets, firms' production decisions are not restricted to entry and exit decisions at the extensive margin and production scale adjustments at the intensive margin. In fact, changes

[7] See Bernard, Redding and Schott, 2011; Eckel and Neary, 2010; Mayer, Melitz and Ottaviano, 2014; and Nocke and Yeaple, 2013.

[8] See Feenstra and Ma, 2007; and Ma, 2009.

[9] See Allanson and Montagna, 2005.

[10] See Baldwin, Caves, Gu, 2005; Bernard, Redding and Schott, 2011; and Mayer, Melitz and Ottaviano, 2013.

in product scope, i.e., in the intra-firm extensive margin, are substantially more frequent than changes in the extensive margin (Bernard, Redding and Schott, 2010; Broda and Weinstein, 2010). Dropping old goods and starting production of new ones are central decisions in firms' production and competition strategy. Bernard, Redding and Schott (2010) find that changes in product scope lead to productivity gains for US manufacturing firms. Product choices are key variables also in strategic actions between firms, with implications on market structure,[11] competition,[12] and incentives to invest in product quality.[13]

An assumption that frequently underlies theoretical studies as well as interpretations of empirical findings is that multiproduct firms conduct flexible manufacturing. Flexible manufacturing means that producers can add new goods to their product assortment without making considerable investments in production technology, albeit the good-specific marginal costs increase as the product scope grows (e.g. Eckel and Neary, 2010). Flexible manufacturing is closely related to the concept of core competency, which means that a multiproduct firm can produce one or a few of its goods more efficiently than the rest of its goods (e.g. Bernard, Redding and Schott, 2011). Production function estimation does not typically accommodate the concepts of flexible manufacturing or core competency, however, apart from a few exceptions discussed below.

Virtually all estimates of production functions are implicitly based on the assumption that all of the firm's output is produced with a firm-level technology.[14] The first set of papers that make an exception evaluate cost minimization with a nonparametric methodology. Cherchye, De Rock and Vermeulen (2008) allow for product-specific technologies as well as economies of scope that result from joint input use and input externalities. Their methodology does not require observable input allocation. Cherchye, De Rock, Dierynck, Roodhooft and Sabbe (2011) build on Cherchye, De Rock and Vermeulen (2008) using a methodology based on data envelopment analysis. In contrast to Cherchye, De Rock and Vermeulen (2008), they use information on output-specific inputs and joint inputs. As a result the discriminatory power of the efficiency measurement is higher, and the efficiency value of the decision making unit can be decomposed into output-specific efficiency values.

[11] See Eaton and Schmitt, 1994.

[12] See Ju, 2003; Johnson and Myatt, 2003, 2006; and Roson, 2012.

[13] See Eckel, Iacovone, Javorcik and Neary, 2011.

[14] There is an early literature on estimating cost functions of multiproduct firms. See, for example, Brown, Caves and Christensen, 1979 and Caves, Christensen and Tretheway, 1980. The early multiproduct cost functions allow for the fact that production technologies across goods vary, but they do not correct the typical endogeneity problems such as the simultaneity or selection bias.

However, the methodology is not suited for any typical firm- or plant-level dataset due to the requirement on observable input allocation. Cherchye, Demuynck, De Rock and De Witte (2011) distinguish between two assumptions: cooperative cost minimization at the firm level, and uncooperative minimization at the level of output department. The advantage of these nonparametric methodologies is that they do not require functional form assumptions. On the other hand, the typical endogeneity biases are not treated.

De Loecker, Goldberg, Khandelwal and Pavcnik (2012) estimate production functions to examine how trade liberalization affects product-specific marginal costs and price markups. They use data on single-product firms and the estimation strategy of Ackerberg, Caves and Frazer (2006) to estimate good-specific production function parameters, which are assumed to be the same for single- and multiproduct firms. In estimating the product-level input allocations De Loecker et al. assume that the share of a firm's materials, labor, and capital allocated to a given product line is constant, i.e., independent of the input type. They show that cost efficiency as well as profitability vary across the various products firms produce. They also find a positive correlation between productivity and the size of the product scope, and suggest that firms may use reductions in marginal costs to finance the development of new products. The method adopted by De Loecker et al. is perhaps closest to the empirical strategy presented in this paper, and the assumptions underlying their estimation method are discussed in section 6.1.

Dhyne, Petrin and Warzynski (2013) study price, markup, productivity and quality dynamics of Belgian manufacturing firms. They modify the proxy approach of Wooldridge (2009) to estimate a product-level production function where the output of a given good is related to the firm-level inputs, the output quantities of the other goods the firm produces, and an unobservable firm-level productivity term. Estimating the production function does not require solving for the unobservable input allocations. However, the output elasticities of the inputs as well as the productivity levels are assumed constant across goods. Dhyne, Petrin and Warzynski also estimate a variable cost function for multiple goods, which takes into account the productivity shocks that are implied by the production function estimates.

3 Model

The model consists of good-specific production and demand functions, and assumptions on the timing of production decisions. Production functions are typically estimated without considering demand for the goods, but in this study output demand is they key for identifying good-specific input allocations and production functions. When firms have market power in the output market, the production decisions are functions of the downward-sloping output demand curves. Functional forms and also most of the other assumptions are familiar from the empirical microeconomic literature. The only exception is that the production function is specified at the product-level instead of the firm-level. The key identifying assumptions are discussed in more detail in sections 4 and 7.

3.1 Production

Firm j produces n_{jt} goods at time t. Production technology i is a good-specific Cobb-Douglas production function with three inputs, materials M_{ijt}, labor L_{ijt}, and capital K_{ijt}:

$$Q_{ijt} = \exp(\beta_{0i}) M_{ijt}^{\beta_{Mi}} L_{ijt}^{\beta_{Li}} K_{ijt}^{\beta_{Ki}} \exp(\omega_{ijt}). \tag{1}$$

Parameters β_{Mi}, β_{Li}, and β_{Ki} denote the output elasticities of materials, labor, and capital for good i, and β_{0i} is a constant. All the production function parameters are good-specific. The productivity term ω_{ijt} varies across goods, firms, and time. It can be divided into expected productivity, $E[\omega_{ijt}|\omega_{ijt-1}]$, and a mean zero productivity shock, ξ_{ijt}:

$$\omega_{ijt} = E[\omega_{ijt}|\omega_{ijt-1}] + \xi_{ijt}. \tag{2}$$

Productivity ω_{ijt} comprises all factors other than M_{ijt}, L_{ijt}, and K_{ijt} that affect the firm's production volume in a given product line and time period. Examples of such factors are management and organization of production and down-time due to, for example, maintenance work and defect rates in the manufacturing process (Ackerberg, Caves and Frazer, 2006). Productivity $\exp(\omega_{ijt})$ is assumed to follow a first-order Markov process. The firm's decision maker forms an expectation of period t's productivity, $E[\omega_{ijt}]$, as a function of the previous period's productivity ω_{ijt-1}. The productivity shock ξ_{ijt} represents a deviation from the expected productivity that takes place or becomes observable at the beginning of period t. The shocks ξ_{ijt} may or may not be correlated across the product lines of the firm. For example, managerial changes may have a similar effect on all the product lines, but they may also have different impacts. Similarly, productivity ω_{ijt} may or may not be correlated across the product lines. The firm may have achieved heterogeneous productivity levels due to, for example, different paths of learning and experience. Also physical economies of scope are captured in the total factor productivity term $\exp(\omega_{ijt})$.

Labor L and capital K are substitutable across the product lines of the firm. All the factors of production are continuously divisible and exclusive across product lines. This means that they can be flexibly allocated across the different product lines, and that any given share of a firm-level input stock is used in only one product line at a time. Furthermore, none of the production functions utilizes other inputs than M_{ijt}, L_{ijt}, and K_{ijt}. This rules out utilization of by-products as factors of production.

3.2 Demand

The firm faces a downward sloping and isoelastic demand curve for each of its goods:

$$Q_{ijt} = \exp(\alpha_{ij})\, P_{ijt}^{\eta_i} \exp(\varepsilon_{ijt}). \tag{3}$$

Price elasticity of demand, η_i, is good-specific and assumed to be lower than -1. Price elastic demand is required to rule out cases where firms produce marginally small output quantities of various goods. The level of demand, denoted by α_{ij}, depends on unobservable factors such as the quality of the good. These factors vary across goods and firms, but they are constant over time. Any shocks to the good- and firm-specific demand level are captured by ε_{ijt}. The shocks can be caused by changes in buyers' preferences or income, prices of substitutes or complementary goods, or the number of buyers in the market, for example.

3.3 Timing of production decisions

The three types of inputs, M_{ijt}, L_{ijt}, and K_{ijt}, differ in how they are determined. The product-level materials M_{ijt} is a flexible input, set or adjusted at the time of production. It is also a static input, meaning that it doesn't have dynamic implications such as adjustment costs. The firm-level human resources[15] L_{jt} and capital stock K_{jt}, on the other hand, are fixed at the time of production, and they are formed in a dynamic process. L_{jt} is chosen in the previous period $t-1$, while the related costs are paid in the period of production. K_{jt} is determined as a function of the previous period's capital stock and investment, $K_{jt} = f(K_{jt-1}, I_{jt-1})$. However, the product-level inputs L_{ijt} and K_{ijt} are allocated across product lines in the period of production, subject to the the firm-level constraints $\sum_i L_{ijt} \leq L_{jt}$ and $\sum_i K_{ijt} \leq K_{jt}$.

The outline of the production decisions is as follows. At time $t-1$, the firm observes its current level of human resources L_{jt-1} and capital stock K_{jt-1}, the expected productivity in product lines i at time t, $E[\omega_{ijt}|\omega_{ijt-1}]$, as well as any other observable factors that affect its future profits. The firm then chooses whether to remain active in period t, and if so, what product titles i to produce. Then, the firm decides on the next period's level of human resources L_{jt} and, by setting the level of capital investment I_{jt-1}, capital stock K_{jt}.

At time t the productivity shocks ξ_{ijt} and the demand shocks ε_{ijt} are realized and become observable to the firm. The firm observes also the price of materials, P_{Mjt}. P_{Mjt} is an exogenous variable, which may reflect the level of bargaining power the firm possesses in the input markets, for example. P_{Mjt} is not a function of the input quantities purchased, however, which implies that there are no cost economies of scope or scale in the form of lower input prices. The firm then chooses the quantities of product-level materials M_{ijt}. At the same time the firm decides how to allocate its human resources L_{jt} and the capital stock K_{jt} among the different product lines the firm is active in, i.e., it sets L_{ijt} and K_{ijt}.

The timing assumptions of this model are similar to the assumptions previously made in the production function literature. These assumptions are compared to those in the previous literature in section 7.

3.4 Firm's optimization problem

The firm maximizes the present discounted value of future profits by making three decisions. First, it chooses which goods i to produce in the next period $t+1$, denoted by

[15] L_{jt} is typically a flexible input in structural production function models. I assume L_{jt} to be fixed because it is more realistic of the Finnish labor market, as discussed in section 7. However, the model can be estimated under either assumption: flexible or fixed labor input.

$D_{ijt+1} = 1$ if it produces good i at $t+1$, and $D_{ijt+1} = 0$ otherwise. Second, the firm decides the human resources L_{jt+1} to be employed in the next period. Third, the firm invests I_{jt} to determine the next period's capital stock K_{jt+1}. These decisions are made given the expected demand and productivity for the goods in the next period, as well the expected future material price.

The Bellman equation for the firm's firm-level dynamic optimization problem is:

$$V\left(S_{jt}\right) = \max_{D_{ijt+1}, L_{jt+1}, I_{jt}} \sum_i \Pi_{ijt}\left(S_{jt}\right) - C\left(I_{jt}\right) + \frac{1}{1+\rho} E[V\left(S_{jt+1}\right)|S_{jt}, D_{ijt}, L_{jt+1}, I_{jt}] \quad (4)$$

where $\Pi\left(S_{jt}\right)$ is the static profit earned in period t, $S_{jt} = \left(\alpha_{ijt}, \eta_{ijt}, \varepsilon_{ijt}, L_{jt}, K_{jt}, \omega_{ijt}, P_{Mjt}\right)$ is the vector of state variables, $C\left(I_{jt}\right)$ is the cost of investment, and ρ is the discount rate. The dynamic optimization problem gives rise to policy functions $D\left(S_{jt}\right)$, $L\left(S_{jt}\right)$ and $I\left(S_{jt}\right)$.

Instead of solving for the dynamic optimization problem,[16] I follow the examples of Olley and Pakes (1996), Levinsohn and Petrin (2003), and Ackerberg, Caves and Frazer (2006), and solve only the static profit maximization problem, which is sufficient for identifying the production function parameters. The static profit maximization problem consists of allocating the firm-level human resources L_{jt} and capital stock K_{jt} among the various product lines i, and setting the product-specific materials M_{ijt} for each product line:

$$\max_{M_{ijt}, L_{ijt}, K_{ijt}} \Pi_{ijt} = \sum_i P_{ijt} Q_{ijt} - P_{Mjt} M_{ijt} \text{ s.t. } \sum_i L_{ijt} \leq L_{jt} \text{ and} \sum_i K_{ijt} \leq K_{jt}. \quad (5)$$

Substituting in the inverse demand, $P_{ijt} = \left(Q_{ijt}\left(\exp(\alpha_{ijt} + \varepsilon_{ijt})\right)^{-1}\right)^{\frac{1}{\eta_{ijt}}}$, as well as the production functions, the static profit maximization problem becomes:

$$\begin{aligned} \max_{M_{ijt}, L_{ijt}, K_{ijt}} \Pi_{ijt} &= \sum_i \left(\exp(\alpha_{ijt} + \varepsilon_{ijt})\right)^{-\frac{1}{\eta_i}} \left(\exp\left(\beta_{0i}\right) M_{ijt}^{\beta_{Mi}} L_{ijt}^{\beta_{Li}} K_{ijt}^{\beta_{Ki}} \exp\left(\omega_{ijt}\right)\right)^{\frac{1}{\eta_i}+1} \\ -P_{Mjt} M_{ijt} \text{ s.t. } \sum_i L_{ijt} &\leq L_{jt} \text{ and} \sum_i K_{ijt} \leq K_{jt}. \end{aligned} \quad (6)$$

The optimization problem yields a Lagrangian equation with two constraints. The constraints account for not exceeding the firm-level human resources L_{jt} and capital stock

[16] Because the dynamic optimization problem is not solved, further specification of the determinants of the dynamic variables is not needed.

K_{jt} when the firm makes input allocations to the product lines. More precisely, given that the firm maximizes profit, L_{jt} and K_{jt} are always fully utilized and the constraints are binding as $\sum_i L_{ijt} = L_{jt}$ and $\sum_i K_{ijt} = K_{jt}$. The Lagrangian is:

$$\begin{aligned} Lagr &= \sum_i \left(\exp(\alpha_{ijt} + \varepsilon_{ijt})\right)^{-\frac{1}{\eta_i}} \left(\exp\left(\beta_{0i}\right) M_{ijt}^{\beta_{Mi}} L_{ijt}^{\beta_{Li}} K_{ijt}^{\beta_{Ki}} \exp\left(\omega_{ijt}\right)\right)^{\frac{1}{\eta_i}+1} \\ &\quad -P_{Mjt}M_{ijt} + \lambda_{Ljt}\left(L_{jt} - \sum_i L_{ijt}\right) + \lambda_{Kjt}\left(K_{jt} - \sum_i K_{ijt}\right). \end{aligned} \tag{7}$$

The first-order conditions for static profit maximization are (JT is the number of firm-time -observations):

$$\begin{aligned} \frac{\partial Lagr}{\partial M_{ijt}} &= \left(\frac{1}{\eta_i}+1\right)\left(\exp(\alpha_{ij} + \varepsilon_{ijt})\right)^{-\frac{1}{\eta_i}} \left(\exp\left(\beta_{0i}\right) M_{ijt}^{\beta_{Mi}} L_{ijt}^{\beta_{Li}} K_{ijt}^{\beta_{Ki}} \exp\left(\omega_{ijt}\right)\right)^{\frac{1}{\eta_i}+1} \frac{\beta_{Mi}}{M_{ijt}} \\ -P_{Mjt} &= 0 \;\forall\; i = [1, n_{jt}] \end{aligned} \tag{8}$$

$$\begin{aligned} \frac{\partial Lagr}{\partial L_{ijt}} &= \left(\frac{1}{\eta_i}+1\right)\left(\exp(\alpha_{ij} + \varepsilon_{ijt})\right)^{-\frac{1}{\eta_i}} \left(\exp\left(\beta_{0i}\right) M_{ijt}^{\beta_{Mi}} L_{ijt}^{\beta_{Li}} K_{ijt}^{\beta_{Ki}} \exp\left(\omega_{ijt}\right)\right)^{\frac{1}{\eta_i}+1} \frac{\beta_{Li}}{L_{ijt}} \\ -\lambda_{Ljt} &= 0 \;\forall\; i = [1, n_{jt}] \end{aligned} \tag{9}$$

$$\begin{aligned} \frac{\partial Lagr}{\partial K_{ijt}} &= \left(\frac{1}{\eta_i}+1\right)\left(\exp(\alpha_{ij} + \varepsilon_{ijt})\right)^{-\frac{1}{\eta_i}} \left(\exp\left(\beta_{0i}\right) M_{ijt}^{\beta_{Mi}} L_{ijt}^{\beta_{Li}} K_{ijt}^{\beta_{Ki}} \exp\left(\omega_{ijt}\right)\right)^{\frac{1}{\eta_i}+1} \frac{\beta_{Ki}}{K_{ijt}} \\ -\lambda_{Kjt} &= 0 \;\forall\; i = [1, n_{jt}] \end{aligned} \tag{10}$$

$$\frac{\partial Lagr}{\partial \lambda_{Ljt}} = L_{jt} - \sum_i L_{ijt} = 0 \;\forall\; jt = [1, JT] \tag{11}$$

$$\frac{\partial Lagr}{\partial \lambda_{Kjt}} = K_{jt} - \sum_i K_{ijt} = 0 \;\forall\; jt = [1, JT]. \tag{12}$$

Although the production functions are product-specific, production of the goods is interdependent because the firm-level human resources and capital stock are fixed at the time of production, and hence the firm has to allocate these inputs across the product lines. The allocation is done as a function of the various demand conditions, production technologies, and the price of materials. Interdependency in production may arise also due to physical economies of scope, which take place when the firm produces several goods and therefore reaches higher productivity levels than when producing only one good.

3.5 Measurement error

The observed variables are product-level Q_{ijt} and P_{ijt}, and firm-level M_{jt}, L_{jt}, K_{jt} and P_{Mjt}. The firm-level materials, M_{jt}, is measured with multiplicative measurement error:

$$\epsilon_{Mjt} = \frac{M_{jt}}{\sum_{i=1}^{n_{jt}} M_{ijt}} - 1. \tag{13}$$

The other observed variables are measured without measurement error.

4 Identification and Estimation Strategy

Firm-level Cobb-Douglas production functions have been estimated in numerous studies. With respect to estimation, the product-specific functions of this paper differ from the firm-level functions in one important aspect: the product-specific inputs are unobservable to the econometrician. This implies that all the elements in the production function are unobservable: input quantities, the output elasticities of the inputs, and total factor productivity. In other words, not only are the the inputs endogenous to the unobservable productivity, which is a standard problem in production function estimation, but they are also unobservable. Clearly, these two problems are closely related.

My identification strategy is based on two insights: one for controlling the endogeneity of inputs to the unobservable productivity level, and another for identifying the unobservable input allocations. The first insight is that, by definition, output is a function of the firm's productivity: the more productive the firm is, the greater its output for any given level of inputs. The unobservable productivity level can be written as a function of the input allocations and the output elasticities of the three inputs, β_{Mi}, β_{Li}, and β_{Ki}. I use this definition of productivity in solving the product-level inputs.

The second insight is that firms make their production decisions as a function of supply-side factors, such as productivity, fixed inputs, and prices of the flexible inputs, but also as a function of the demand for the goods. Intuitively, the higher the demand for a given good, the more inputs the firm is willing to allocate to the product line. Shocks in output demand provide a source of variation for identifying the optimal input allocations.

Furthermore, as an overidentifying assumption I can use the notion that the product-level inputs estimated sum up to the observable firm-level inputs.

The optimal input choices are solved analytically from the firm's static profit maximization problem, as a function of the productivity term ω_{ijt} and up to the production function parameters β_{0i}, β_{Mi}, β_{Li} and β_{Ki} (recall that the state variables $S_{jt} = (\alpha_{ijt}, \eta_{ijt}, \varepsilon_{ijt}, L_{jt}, K_{jt}, \omega_{ijt}, P_{Mjt})$):

$$M_{ijt} = f_M\left(S_{ijt}, \beta_{0i}, \beta_{Mi}, \beta_{Li}, \beta_{Ki}\right) \tag{14}$$

$$L_{ijt} = f_L\left(S_{ijt}, \beta_{0i}, \beta_{Mi}, \beta_{Li}, \beta_{Ki}\right) \tag{15}$$

$$K_{ijt} = f_K\left(S_{ijt}, \beta_{0i}, \beta_{Mi}, \beta_{Li}, \beta_{Ki}\right). \tag{16}$$

As explained above, the first key of the estimation strategy is using the definition of the productivity term ω_{ijt} in controlling for the endogeneity of inputs. Inverting the production function for ω_{ijt}, I get:

$$\omega_{ijt} = \log\left(\frac{Q_{ijt}}{\exp(\beta_{0i}) M_{ijt}^{\beta_{Mi}} L_{ijt}^{\beta_{Li}} K_{ijt}^{\beta_{Ki}}}\right). \tag{17}$$

By substituting this definition of ω_{ijt} in the analytical input functions $M_{ijt}, L_{ijt}, K_{ijt}$, I obtain:

$$M'_{ijt} = g_M\left(S'_{ijt}, Q_{ijt}, \beta_{0i}, \beta_{Mi}, \beta_{Li}, \beta_{Ki}\right) \tag{18}$$

$$L'_{ijt} = g_L\left(S'_{ijt}, Q_{ijt}, \beta_{0i}, \beta_{Mi}, \beta_{Li}, \beta_{Ki}\right) \tag{19}$$

$$K'_{ijt} = g_K\left(S'_{ijt}, Q_{ijt}, \beta_{0i}, \beta_{Mi}, \beta_{Li}, \beta_{Ki}\right), \tag{20}$$

where S'_{ijt} denotes the state variables without ω_{ij}. By imposing $M'_{ijt} = M_{ijt}$, $L'_{ijt} = L_{ijt}$, and $K'_{ijt} = K_{ijt}$, and substituting $M'_{ijt}, L'_{ijt}, K'_{ijt}$ and the definition of ω_{ijt} in the production function, I take account of the unobservable productivity level. The production function for good i can then be written as:

$$Q_{ijt} = \exp\left(\beta_{0i}\right) M_{ijt}^{\beta_{Mi}} L_{ijt}^{\beta_{Li}} K_{ijt}^{\beta_{Ki}} \exp\left(\omega_{ijt}\right), \tag{21}$$

where β_{0i}, β_{Mi}, β_{Li} and β_{Ki} are the only unobservables. But when written in this form, an infinite number of parameters β_{0i}, β_{Mi}, β_{Li} and β_{Ki} solve the empirical production function. This is because ω_{ijt} is inverted from the production function itself. However, the production function can be identified using the structure of the productivity process, which is a function of the expectation of productivity $E[\omega_{ijt}|\omega_{ijt-1}]$, and the productivity shock ξ_{ijt}.

Using the productivity shock ξ_{ijt} in identification is a standard practice in structural production function models (Olley and Pakes, 1996; Levinsohn and Petrin, 2003; Ackerberg, Caves and Frazer, 2006). Lagged static inputs, in this paper M_{ijt}, are correlated over time but uncorrelated with the productivity shock. Fixed inputs, in this case L_{ijt} and K_{ijt}, are chosen prior to observing ξ_{jt}. Hence, they are not correlated with the productivity shock. As the fixed inputs L_{ijt} and K_{ijt} are subject to different input costs, the two variables are not collinear.

Given the standard assumptions I make regarding the timing of input choices, and given that there are sufficiently many sources of identifying variation, the above moments can be modified to suit the production function specified in this paper. The productivity shocks only have to be specified at the product-level:

$$E\left[\xi_{ijt}|M_{jt-1}\right] = 0 \;\forall\; i = [1, N] \quad (22)$$

$$E\left[\xi_{ijt}|L_{jt}\right] = 0 \;\forall\; i = [1, N] \quad (23)$$

$$E\left[\xi_{ijt}|K_{jt}\right] = 0 \;\forall\; i = [1, N]. \quad (24)$$

The firm-level M_{jt-1}, L_{jt}, and K_{jt} are correlated with the product-level M_{ijt}, L_{ijt}, and K_{ijt} because the firm-level variables are sums of the product-level inputs. An additional instrument is the price of the flexible input, correlated with M_{ijt} but uncorrelated with ξ_{ijt}:

$$E\left[\xi_{ijt}|P_{Mjt}\right] = 0 \;\forall\; i = [1, N]. \quad (25)$$

P_{Mjt} is a valid instrument even if measured with error because the measurement error is not correlated with the productivity shock.

Demand for good i would also be a valid instrument. Demand for good i correlates positively with the input choices M_{ijt}, L_{ijt} and K_{ijt}, while it is uncorrelated with the productivity shock ξ_{ijt}. Unfortunately, the demand is unobservable. However, the output prices are informative about the underlying demand. Price for good i depends on the output quantity produced and the level of productivity at which it is produced, that is, P_{ijt} is correlated with the productivity shock and hence not a valid instrument. However, lagged price P_{ijt-1} is correlated with the demand for good i also at time t, and hence with the input choices M_{ijt}, L_{ijt} and K_{ijt}, because demand for good i is correlated over time as denoted by α_{ij}. At the same time, P_{ijt-1} is uncorrelated with the productivity shock:

$$E\left[\xi_{ijt}|P_{ijt-1}\right] = 0 \;\forall\; i = [1, N]\,. \tag{26}$$

I also use the fact that product-level inputs M_{ijt} add up to the firm-level input M_{jt}, which is observable but measured with measurement error. Any firm-level measurement error in M_{jt}, denoted by ϵ_{Mjt}, is expected to be zero. A valid instrument for identifying β_{Mi} is the product of output price and quantity, $P_{ijt}Q_{ijt}$, which is uncorrelated with the measurement error in materials ϵ_{Mjt}, but correlated with the use of materials M_{ijt}:

$$E\left[\epsilon_{Mjt}|P_{ijt}Q_{ijt}\right] = 0 \;\forall\; i = [1, N]\,. \tag{27}$$

These moment conditions identify the production technologies.

Identification of the demand functions requires an instrument[17] for the endogeneous prices. The material price P_{Mjt}, human resources L_{jt}, and capital stock K_{jt} correlate with the product prices but they are uncorrelated with the product- and firm-specific demand shocks ε_{ijt}:

$$\begin{aligned} E\left[\varepsilon_{ijt}|P_{Mjt}\right] &= 0 \;\forall\; i = [1, N] && (28)\\ E\left[\varepsilon_{ijt}|L_{jt}\right] &= 0 \;\forall\; i = [1, N] && (29)\\ E\left[\varepsilon_{ijt}|K_{jt}\right] &= 0 \;\forall\; i = [1, N]\,. && (30) \end{aligned}$$

The model is identified with these moments and estimated by GMM.

[17] For a discussion on instruments used in demand estimation, see, for example, Ackerberg, Benkard, Berry and Pakes, 2007.

4.1 Solving for ξ_{ijt}, ε_{ijt}, and ϵ_{Mjt}

The productivity shock ξ_{ijt} is:

$$\xi_{ijt} = \log\left(\frac{Q_{ijt}}{\exp\left(\beta_{0i}\right) M_{ijt}^{\beta_{Mi}} L_{ijt}^{\beta_{Li}} K_{ijt}^{\beta_{Ki}}}\right) - E\left[\omega_{ijt}|\omega_{ijt-1}\right] \tag{31}$$

where M_{ijt}, L_{ijt}, K_{ijt} and $E\left[\omega_{ijt}|\omega_{ijt-1}\right]$ are unknown. M_{ijt}, L_{ijt}, and K_{ijt} are solved from the first-order conditions for static profit maximization, the definition of productivity for the estimation equation, $\omega_{ijt} = \log\left(Q_{ijt}(\exp\left(\beta_{0i}\right) M_{ijt}^{\beta_{Mi}} L_{ijt}^{\beta_{Li}} K_{ijt}^{\beta_{Ki}})^{-1}\right)$, and the demand function inverted for price, $P_{ijt} = \exp\left(\alpha_{ij} + \varepsilon_{ijt}\right)^{-\frac{1}{\eta_i}} Q^{\frac{1}{\eta_i}}$. By substitution:

$$\begin{aligned}
M_{ijt} &= \left(\frac{1}{\eta_{ijt}} + 1\right) P_{ijt} Q_{ijt} \frac{\beta_{Mi}}{P_{Mjt}} \;\forall\; i = [1, n_{jt}] && (32)\\
L_{ijt} &= \frac{\left(\frac{1}{\eta_{ijt}} + 1\right) P_{ijt} Q_{ijt} \beta_{Li} L_{jt}}{\sum_i \left(\frac{1}{\eta_{ijt}} + 1\right) P_{ijt} Q_{ijt} \beta_{Li}} \;\forall\; i = [1, n_{jt}] && (33)\\
K_{ijt} &= \frac{\left(\frac{1}{\eta_{ijt}} + 1\right) P_{ijt} Q_{ijt} \beta_{Ki} K_{jt}}{\sum_i \left(\frac{1}{\eta_{ijt}} + 1\right) P_{ijt} Q_{ijt} \beta_{Ki}} \;\forall\; i = [1, n_{jt}]\,. && (34)
\end{aligned}$$

Given M_{ijt}, L_{ijt}, K_{ijt}, and the implied ω_{ijt}, the productivity process is estimated with the following estimation equation:

$$\omega_{ijt} = g\left(\omega_{ijt-1}\right) + \xi_{ijt} \tag{35}$$

where $g\left(\omega_{ijt-1}\right)$ is a second-order polynomial of the lagged productivity term $\omega_{ijt-1}\left(\beta_{Mi}, \beta_{Li}, \beta_{Ki}\right)$, and ξ_{ijt} is the productivity shock.[18]

Given the solution for M_{ijt} (32), the multiplicative input measurement error ϵ_{Mjt} is computed as:

$$\epsilon_{Mjt} = \frac{M_{jt}}{\sum_{i=1}^{n_{jt}} M_{ijt}} - 1. \tag{36}$$

[18] The parameters in the polynomial $g\left(\omega_{ijt-1}\right)$, denoted by γ_i, enter the moment conditions linearly. Hence they can be concentrated out from the estimation routine for the nonlinear parameters. The linear parameters γ_i are obtained by regressing the productivity level implied by a given set of parameter values $\omega_{ijt}\left(\beta'_{Mi}, \beta'_{Li}, \beta'_{Ki}\right)$ on the second-order polynomial terms of the implied lagged productivity $\omega_{ijt-1}\left(\beta'_{Mi}, \beta'_{Li}, \beta'_{Ki}\right)$.

The demand shock ε_{ijt} is:

$$\varepsilon_{ijt} = \log\left(\frac{Q_{ijt}}{\exp(\alpha_{ij})P_{ijt}^{\eta_i}}\right) \tag{37}$$

where the unobservable product-firm -specific demand level, α_{ij} is (T_{ij} is the number of time periods in which firm j has produced good i):

$$\alpha_{ij} = T_{ij}^{-1}\sum_{t=1}^{T_{ij}}\log\left(\frac{Q_{ijt}}{P_{ijt}^{\eta_i}}\right). \tag{38}$$

5 Data and Empirical Implementation

5.1 Data

I use the Longitudinal Database on Plants in Finnish Manufacturing (LDPM) and the Industrial output data of Statistics Finland on years 2004 - 2011. The two datasets include plants that belong to manufacturing firms with at least 20 employees, and a subset of plants of firms with less than 20 employees. The reporting units are mainly plants. The only exceptions are in the Industrial output data, where a few plants belonging to the same firm report jointly. For these reporting units I aggregate the observations in the LDPM accordingly.

I estimate the production functions of firms in Division 16, "Manufacture of wood and of products of wood and cork, except furniture; manufacture of articles of straw and plaiting materials". The products are classified according to Eurostat's 8-digit PRODCOM (Production communautaire) codes that are supplemented by national 10-digit subclasses. Goods within the fairly narrowly defined titles are therefore comparable in physical quantities. The titles are provided in Table 1. For each product title a plant produces in a given year, I observe the output measured in a physical unit as well as the sales revenue. These two yield the average price of the good in the given year. Similarly for the intermediate products and materials I observe physical quantities and expenditures by the PRODCOM titles. The "price" of materials is computed as the Elteto-Koves-Szulc (EKS) multilateral price index (see, for example, Hill, 2004, and Neary, 2004). For firm a it can be expressed as follows:

$$P^a_{EKS} = \prod_{j=1}^{J} \left(\frac{P_F\left(q^j, q^a, p^j, p^a\right)}{P_F\left(q^j, q^b, p^j, p^b\right)} \right)^{\frac{1}{J}}, \tag{39}$$

where q^j and p^j are the quantity and price vectors of firm j, and $P_F\left(q^j, q^a, p^j, p^a\right)$ is the bilateral Fisher price index between firm a and firm j, $j = 1, ..., J$ (J is the number of firms), which is given by

$$P_F\left(q^j, q^a, p^j, p^a\right) = \left(\frac{q^j * p^a}{q^j * p^j} * \frac{q^a * p^a}{q^a * p^j} \right)^{\frac{1}{2}}, \tag{40}$$

where $q^j * p^j = \sum_{n=1}^{N} q^j_n p^j_n$ (N is the number of product titles). Similarly for $P_F\left(q^j, q^b, p^j, p^b\right)$, where b stands for the base firm chosen. The EKS multilateral index satisfies the circularity (transitivity) requirement, which implies that the same index is obtained irrespective of whether firms are compared with each other directly, or through their relationships with other firms (Hill, 2004; Neary, 2004). The EKS multilateral index is thus well-suited for my purpose of comparing firms when no representative firm exists, and bundles of goods differ between firms.

The labor input is measured in labor costs that comprise salary and social payments. The monetary value of the capital stock is estimated using the perpetual inventory method, $K_{jt} = \delta K_{jt-1} + I_{jt-1}$, where $\delta = 0.9$ and I_{jt} is investment.

The estimation methodology poses certain requirements on the observations. First, all product titles need to be observed in at least four pairs of observations, each pair being from two consecutive years in a given firm. This is because for each product title there are four non-linear parameters to be estimated, and because estimating the 1st order Markov process of productivity evolution requires sequences of at least two observations. Second, observations with missing variables cannot be used in estimation. Observations that do not fulfill the aforementioned criteria are dropped from the sample.

Note that measurement error in output is assumed to be zero. Unfortunately, there is no other output variable that could be used to verify the accuracy of the product-specific sales revenue variables. The only other output variable available is the plant-level gross output reported in the LDPM. Gross output is defined as the sum of sales revenue, deliveries to other plants of the firm, changes in inventories, production for own use, and other business revenue, deducting capital gains and acquisition of merchandise. Not suprisingly, gross output is not equal to the sum of product-specific sales revenues from production in all of the plants. As the definition of gross output goes, there are several potential explanations for this. Plants may produce output that is not included in the sales revenue from production (deliveries to other plants of the firm, positive changes in inventories, production for own use), or the sales revenue data may include output produced in some previous year (negative changes in inventories). Moreover, because capital gains and acquisition of merchandise are deducted from gross output, it is not possible to make strong inferences about potential measurement error in output. Unfortunately, the various components of gross output are not reported in the LDPM, and hence I cannot identify why gross output may differ from sales revenue. However, to reduce the likelihood of using observations with major measurement error in output, I use only those observations for which the ratio of sum of sales revenue to gross output is at least 0.6 but not more than 1.4.

In the final sample there are 2053 good-plant-year -level observations and 904 plant-year -level observations, collected from 190 plants during 8 years. In total, 42 different product titles are produced. Plants' product assortments range from 1 up to 17 product titles. A plant produces on average 3.25 product titles.

5.2 Product line specification

Every product title i is related to four nonlinear parameters that need to be estimated: price elasticity η_i, and output elasticities β_{Mi}, β_{Li} and β_{Ki}. If I defined the parameters at the 8- or 10-digit level, I would need to estimate $42 \times 4 = 168$ nonlinear parameters. At least in my setting this is a too large a number of nonlinear parameters to be estimated. Instead, I define the parameters at the 3-digit level, which yields two product categories: "Sawmilling and planing of wood" (PRODCOM code 161), and "Manufacture of products of wood, cork, straw and plaiting materials" (162). This specification implies estimating $2 \times 4 = 8$ nonlinear parameters. The parameters governing the productivity process $g\left(\omega_{ijt-1}\right)$ are also specified at the 3-digit level. The constants β_{0i} are specific to the goods as defined at the 8- or 10-digit level. Also the productivity levels ω_{ijt} and the productivity shocks ξ_{ijt} are specific to the 8- or 10-digit titles.

There are 15 product titles in category 161, and 27 titles in category 162. A plant produces on average 2.17 titles in category 161, and 1.08 titles in category 162. 56% of the plants in the sample produce at least one good in category 161, and 61% of the plants produce at least one good in category 162. 86% of the plants that produce any good in category 161 produce at least two titles in that category. Similarly, 43% of the plants that produce any good in category 162 produce more than one title in that category.

5.3 Optimal instruments

To improve the estimator's efficiency, I replace some of the moment conditions discussed above by moments with optimal instruments. Amemiya (1974) derives optimal instruments for non-linear models, and Arellano (2003) provides an overview of optimal instruments in linear and nonlinear models. Reynaert and Verboven (2012) show that adopting Chamberlain's (1987) optimal instruments in estimating the randon coefficients logit demand model of Berry, Levinsohn, Pakes' (1995) reduces the small sample bias and increases the estimator's efficiency and stability.

The optimal instrument is the expected value of the derivative of the structural error term with respect to the parameter, computed at an initial estimate of the parameters:

$$z_{ijt} = E\left[\frac{\partial \xi_{ijt}(\theta)}{\partial \theta'} \mid X_{ijt}\right] \tag{41}$$

where θ contains the parameters to be estimated, $\theta = (\eta, \beta, \gamma)$, and X_{ijt} comprises the observables, $X_{ijt} = (Q_{ijt}, P_{ijt}, P_{Mjt}, L_{jt}, K_{jt})$. Because the optimal instruments are non-linear functions of the parameters to be estimated, they cannot be computed directly from the data. Instead the optimal instruments are updated after each stage of GMM. In the first stage I use starting values that are an educated guess of the parameters. For the subsequent rounds, the optimal instruments are recomputed using the parameter estimates from the previous stage of GMM.

I replace all the supply-side moments with productivity shocks ξ_{ijt} and standard instruments by moments with optimal instruments. As compared to the empirical model with standard instruments, the objective function appears smoother, and the estimates less responsive to the starting values. This is because the functional forms imposed are exploited to a fuller extent.

I do not adopt optimal instruments for the other moments, i.e., the moments that contain the measurement error ϵ_{Mjt} or demand shock ε_{ijt}. The reason is that writing optimal instruments when the structural error term is a function of endogenous observations is complicated (Arellano 2003). In summary, the moment conditions I use are:

$$\begin{array}{cc} \text{Moment} & \text{Parameter to be identified} \\ E\left[\xi_{ijt}|z_{Mijt}\right] = 0 \;\forall\; i = [1, N] & \beta_{Mi} \\ E\left[\xi_{ijt}|z_{Lijt}\right] = 0 \;\forall\; i = [1, N] & \beta_{Li} \\ E\left[\xi_{ijt}|z_{Kijt}\right] = 0 \;\forall\; i = [1, N] & \beta_{Ki} \\ E\left[\epsilon_{Mjt}|P_{ijt}Q_{ijt}\right] = 0 \;\forall\; i = [1, N] & \beta_{Mi} \\ E\left[\varepsilon_{ijt}|P_{Mjt}\right] = 0 \;\forall\; i = [1, N] & \eta_i \\ E\left[\varepsilon_{ijt}|L_{jt}\right] = 0 \;\forall\; i = [1, N] & \eta_i \\ E\left[\varepsilon_{ijt}|K_{jt}\right] = 0 \;\forall\; i = [1, N] & \eta_i \end{array} \tag{42}$$

As four moment conditions are sufficient for exact identification of the model, there are three overidentifying restrictions in the above set of moments. Some of the 8- or 10-digit product titles have at least four but less than seven observation pairs. In these cases I cannot use all the seven moment conditions. Instead of dropping observations of the product title entirely, I drop some of the overidentifying moments for these products. For product i with only four observations pairs, I adopt moments $E\left[\xi_{ijt}|z_{Mijt}\right] = 0$, $E\left[\xi_{ijt}|z_{Lijt}\right] = 0$, $E\left[\xi_{ijt}|z_{Kijt}\right] = 0$, and $E\left[\varepsilon_{ijt}|P_{Mjt}\right] = 0$. Moment $E\left[\epsilon_{Mjt}|P_{ijt}Q_{ijt}\right] = 0$ ($E\left[\varepsilon_{ijt}|L_{jt}\right] = 0$) [$E\left[\varepsilon_{ijt}|K_{jt}\right] = 0$] is used when there is at least five (six) [seven] observation pairs.

Estimates of the production function parameters β_{Mi}, β_{Li} and β_{Ki} and the price elasticities η_i are obtained by iterated GMM.

6 Results

As there are multiple parameters to be estimated that enter the GMM objective function non-linearly, finding the global minimum can be challenging. To make sure that the estimation routine reaches the global minimum of the GMM objective function, I experiment with various minimization algorithms, of which the Gauss-Newton algorithm turns out to perform best in finding the global minimum among the local minima. I also run the estimation routine with a large set of alternative starting values.[19]

[19] The starting values for β_{Mi}, β_{Li} and β_{Ki} range between 0.15 and 0.5, and the starting values for η_i between -8 and -1.5.

The estimation results are presented in Table 2. The two production functions and demand functions estimated are for two groups: "Sawmilling and planing of wood" (PRODCOM titles 161), and "Manufacture of products of wood, cork, straw and plaiting materials" (PRODCOM titles 162). All the non-linear parameter estimates are statistically significant.[20] Also, the estimates of the output elasticities are statistically different for the technologies of the two product groups. The output elasticity of materials is considerably higher in the technology for titles 162 than in the technology for 161 (β_M for 162 is 0.74 and β_M for 161 is 0.38). The output elasticity of labor, again, is considerably lower in the technology for titles 162 (β_L for 162 is 0.12 and β_L for 161 is 0.35). Both technologies have output elasticity of capital of the same magnitude (β_K for 161 is 0.19 and β_K for 162 is 0.18). Returns to scale are different for the two technologies: the technology for product titles 161 is subject to decreasing returns to scale ($\beta_M + \beta_L + \beta_K = 0.93 < 1$) , while the technology for titles 162 has increasing returns to scale ($\beta_M + \beta_L + \beta_K = 1.04 > 1$). In short, the various goods in the product groups "Sawmilling and planing of wood" and "Manufacture of products of wood, cork, straw and plaiting materials", which many multiproduct firms simultaneously produce, are not manufactured with a single firm-level production technology.

The demand for titles 161 is more price elastic than the demand for titles 162, as η for titles 161 is -1.30 and η for titles 162 is -1.12. This is intuitive because products of wood, cork, straw and plaiting materials are likely to be more differentiated than the output of sawmilling and planing of wood. Hansen's J-test does not reject the null hypothesis of valid overidentification restrictions (Prob[Chi-sq.(264)>J] is 0.4632).

7 Discussion on Identification

The structural production function literature focuses on correcting for endogeneity biases. Several papers build on the insight of Olley and Pakes (1996) that because inputs are set as a function of the firm's productivity, input demand can be inverted for the unobservable productivity term. Subsequently this idea, referred to as the proxy method, has been used by Levinsohn and Petrin (2003), Ackerberg, Caves and Frazer (2006), Wooldridge (2009), and Doraszelski and Jaumandreu (2013). Gandhi, Navarro and Rivers (2013) use

[20] The product-firm specific demand levels α_{ij}, the 42 constants β_{0i}, and the parameters governing the productivity process $g(\omega_{ijt-1})$ are not reported.

firms' short run first order conditions to control for the collinearity of inputs. Most of the assumptions underlying my identification strategy are familiar from this literature. I make also some novel assumptions, and relax some of the assumptions previously made.

All the moment conditions, in my and other structural production function estimation strategies, are based on assumptions about the timing of input choices with respect to productivity shocks. In addition, I specify the role of demand shocks in production choices. Materials M_{ijt} are chosen only after the demand and productivity shocks ε_{ijt} and ξ_{ijt} have been observed, while the firm-level labor L_{jt} and capital stock K_{jt} are determined before the shocks. These assumptions are standard in the literature, apart from taking account of the demand shocks in production decisions, and assuming L_{jt} to be a fixed variable. The reason for treating L_{jt} as a fixed input is not technical, but this assumption is made to account for the environment in which the data has been generated: employment protection legislation plays a significant role in Finland. The OECD indicators of employment protection (OECD, 2013) measure the strictness of legislation on individual and collective dismissals and the strictness of hiring employees on temporary contracts. The measures are based on information about statutory and case laws, collective bargaining agreements, and advice by officials from OECD member countries and country experts. According to these indicators, the Finnish labor market was of the OECD average in the strictness of employment protection during the period of 2004 to 2011. Based on this measure, fixed labor input is a realistic assumption. In case the method of this paper is to be used for estimating production functions in an economy where flexible labor input is a more appropriate assumption, the empirical model can be adjusted accordingly. As in other structural production function models, one flexible input is required for inverting out the unobservable productivity ω_{ijt}. I also further specify that the product-level labor and capital allocations L_{jt} and K_{ijt} are set as endogeneous to ε_{ijt} and ξ_{ijt}. This assumption not only facilitates the estimation of L_{jt} and K_{ijt}, but also allows firms to reallocate human resources and capital as response to demand and productivity shocks.

An important difference in the timing assumptions of this and other structural estimation strategies is that I assume away any productivity shocks once the flexible inputs have been set, and measurement error in output Q_{ijt}. I make these assumptions in order to solve for the unobservable input allocations, while controlling for the unobservable productivity ω_{ijt}. At the same time, and in contrast to the rest of the literature, I allow for measurement error in the flexible inputs M_{jt} observed at the firm-level. This provides me an additional moment condition for identifying β_{Mi}, as compared to the other production models: sales revenue from a given product correlates positively with the flexible input M_{ijt} allocated to the product line, but is uncorrelated with the firm-level measurement error in M_{jt}, denoted by ϵ_{Mjt}.

In addition to the timing assumptions, the proxy methods require two more key assumptions. First, input demand is assumed monotonic in productivity. In other words, cases where input demand may decrease due to improved efficiency are assumed away. However, this assumption may be unrealistic in settings where firms face downward sloping demand curves. I relax the monotonicity assumption by using the definition of productivity itself in controlling for endogeneity.

Second, the proxy methods require the assumption that productivity ω_{ijt} is the only scalar unobservable that affects the input choices. Unobservable inter-firm variation in, say, input prices or output demand, as well as optimization and measurement error in the flexible inputs, are assumed away. I also need to make the scalar unobservability assumption for estimating product-level inputs. However, I do allow for measurement error in the flexible inputs. I also allow for inter-firm variation in input prices and output demand. In fact, I need input prices and estimates of output demand for estimating the input allocations. At the same time, variation in the input prices resolves the collinearity problem between the flexible input M_{ijt} and the other inputs. What the scalar unobservability assumption in my application implies is that the price a firm pays for its flexible input, P_{Mjt}, does not depend on the quantity purchased M_{ijt}. By modelling supply in the input market this assumption could be relaxed, however. As in other empirical strategies, I also assume that the input demand function is continuous. In other words, firms can purchase precisely the input quantity that maximizes their profit. This seems justified after eyeballing the firm-level input data.

The last set of supply-side assumptions that I make concerns the inputs. Units of the firm-level input stocks L_{jt} and K_{jt} are substitutable between product lines, and there are no adjustment costs in (re)allocating labor or capital to other product lines. Also, a firm does not use production of a given good as an input for another good. In fact, these assumptions are not specific to this product-specific model, but they are made implicitly in all firm-level estimations when firms produce more than one type of good.

In contrast to the other structural methods, the one of this paper requires demand estimates for identifying the unobservable input allocations. Identification of the demand function is based on two assumptions. First, any unobservables that affect the demand for a given good of a given firm, e.g. product quality, are constant over time. This assumption may be realistic for some industries, and unrealistic for others. If unrealistic, the demand model can be replaced with a more flexible one. Second, changes in input prices and fixed input stocks shift the supply curve, while the demand curve, including the demand shock ε_{ijt}, is not affected. Using material prices and fixed input stocks as instruments is a standard practice. Also note that the estimated product-level inputs M_{ijt}, L_{ijt}, and K_{ijt} enter the production function as generated regressors. In order for the production function estimates to be consistent, all the instruments, generated and observed, need to be uncorrelated with the residuals (Wooldridge, 2002). In other words, if the moment conditions are valid, the parameter estimates are consistent.

To sum up, recall that the estimation biases acknowledged in the literature are: selection, simultaneity, collinearity, omitted price, and product bias, as discussed in section 2. The estimation strategy of this paper does not consider the selection bias.[21] Nevertheless, it is possible to extend the strategy to control for market entry and selection to various product lines by computing propensity scores for market entry, as in Olley and Pakes (1996). Furthermore, the selection bias may be less of a problem when product-level capital is a quasi-flexible variable, i.e., capital allocations to product lines are made in the period of production given a fixed firm-level capital stock. Recall that the selection bias arises due to a negative correlation between firms' capital stock and productivity level in the sample. But when capital allocations to product lines are set as a function of productivity and demand, as in the multiproduct case, it is not obvious whether the correlation between capital and productivity is positive or negative. Hence identifying β_{Ki} is now potentially subject to two opposing biases: selection bias (towards zero), and simultaneity bias (away from zero). The simultaneity bias of β_{Ki} is corrected as the biases of β_{Li} and β_{Mi}. The selection bias is not corrected for, but the problem is alleviated due to allocation of capital across product lines.

The other four of the five biases are accounted for. The simultaneity bias is corrected

[21] In fact, the method of Olley and Pakes (1996) is the only one that corrects for the selection problem, while the other structural methods focus on accounting for the simultaneity problem.

by writing input functions explicitly as a function of the unobservable productivity. Identifying variation in material prices and fixed inputs stocks resolves the collinearity problem. The omitted price bias doesn't occur because input and output prices are observed, and physical quantity measures are used instead of sales revenues and input expenditures. The so-called product bias is corrected by allowing for good-specific production technology, and by taking account of the role of output demand in production decisions.

The identification strategy accommodates also other functional forms than the Cobb-Douglas production function and the isoelastic demand function used in this paper. The requirement on the production model, as in most structural production models, is that there has to be at least one input that is chosen as a function of the unobservable productivity. The data is required to include observations of at least two consecutive periods, and report physical output and sales revenue by product title. Such data, fortunately, is provided by many national statistical offices in Europe, for example.

7.1 Comparison with De Loecker et al.

There are a few recent papers that also accommocate for multiproduct firms and product-specific production technologies, as mentioned in the literature review. The method of De Loecker, Goldberg, Khandelwal and Pavcnik (2012, henceforth DLGKP) is perhaps closest to the method presented in this paper. DLGKP and I have rather similar datasets where input allocations within firms are unobservable. We also make many similar identifying assumptions that are standard in the structural production function literature, as DLGKP use the empirical model and estimation strategy of Ackerberg, Caves and Frazer (2006). Nevertheless, our key assumptions and empirical strategies that address the unobservable input allocations are quite different.

Both DLGKP and I assume that single- and multiproduct firms use similar product-specific technologies. DLGKP are able to utilize this assumption to a fuller extent because they observe sufficiently many single-product firms to estimate the technology parameters using data on those firms only. This enables DLGKP to estimate the parameters without simultaneously solving for the unobservable input allocations. The input allocations are computed using the parameter estimates and the observable variables. DLGKP assume that the share of a firm's materials, labor, and capital allocated to a given product line is constant, i.e., independent of the input type. This implies that a firm produces all of its goods with the same materials-labor-capital -ratio. However, a profit maximizing or cost minimizing firm would not allocate inputs to product lines with such constant ratios. Even when the technology parameters are correctly estimated, estimates of the unobservable productivity levels are affected by this assumption. On the other hand, DLGKP avoid making the assumption of zero productivity shocks after the flexible inputs have been set, which I need to make. Moreover, DLGKP do not require estimates of output demand.

8 Conclusion

This paper contributes to a large empirical literature on production function estimation, which underlies even a larger body of applied economic research. To take account of the empirical fact that a remarkable share of firms is multiproduct firms, I provide a method to estimate product-specific production functions when some or all firms produce multiple goods. The method does not require data on input allocations to various product lines. Instead, output demand is estimated to identify the input allocations to the product lines and the production functions. Endogeneity of the input allocations to the unobservable productivity levels is controlled for by using the inverses of the production functions in solving for the input allocations. The method is demonstrated by estimating production functions for goods in the industry "Manufacture of wood and of products of wood and cork, except furniture; manufacture of articles of straw and plaiting materials". I find that the technologies used in "Sawmilling and planing of wood" (PRODCOM 161) and "Manufacture of products of wood, cork, straw and plaiting materials" (PRODCOM 162) are statistically different from each other. The empirical findings suggest that production functions should be estimated at the product- instead the firm-level, and that multiproduct firms use multiple production technologies.

References

Ackerberg, D., L. C. Benkard, S. Berry, and A. Pakes (2007): *Econometric Tools for Analyzing Market Outcomes*vol. 6 of *Handbook of Econometrics*, chap. 63. Elsevier, 1 edn.

Ackerberg, D., K. Caves, and G. Frazer (2006): "Structural identification of production functions," MPRA Paper 38349, University Library of Munich, Germany.

Ackerberg, D., G. S. Crawford, and J. Hahn (2011): "Orthogonal Instruments: Estimating Price Elasticities in the Presence of Endogenous Product Characteristics," Slides on Crawford's website.

Allanson, P., and C. Montagna (2005): "Multiproduct firms and market structure: An explorative application to the product life cycle," *International Journal of Industrial Organization*, 23(7-8), 587–597.

Amemiya, T. (1977): "The Maximum Likelihood and the Nonlinear Three-Stage Least Squares Estimator in the General Nonlinear Simultaneous Equation Model," *Econometrica*, 45(4), 955–968.

Arellano, M. (2003): *Panel Data Econometrics.* Oxfor University Press.

Baldwin, J. R., R. Caves, and W. Gu (2005): "Responses to Trade Liberalization: Changes in Product Diversification in Foreign- and Domestic-controlled Plants," Economic Analysis (EA) Research Paper Series 31, Statistics Canada, Analytical Studies Branch.

Berger, A. N., and T. H. Hannan (1998): "The Efficiency Cost of Market Power In The Banking Industry: A Test Of The "Quiet Life" And Related Hypotheses," *Review of Economics and Statistics*, 80(3), 454–465.

Bernard, A. B., J. B. Jensen, S. J. Redding, and P. K. Schott (2007): "Firms in International Trade," *Journal of Economic Perspectives*, 21(3), 105–130.

Bernard, A. B., S. J. Redding, and P. K. Schott (2009): "Products and Productivity," *Scandinavian Journal of Economics*, 111(4), 681–709.

——— (2010): "Multiple-Product Firms and Product Switching," *American Economic Review*, 100(1), 70–97.

——— (2011): "Multiproduct Firms and Trade Liberalization," *Quarterly Journal of Economics*, 126(3), 1271–1318.

Bond, S., and M. Söderbom (2005): "Adjustment costs and the identification of Cobb Douglas production functions," IFS Working Papers W05/04, Institute for Fiscal Studies.

Broda, C., and D. E. Weinstein (2010): "Product Creation and Destruction: Evidence and Price Implications," *American Economic Review*, 100(3), 691–723.

Brown, R. S., D. W. Caves, and L. R. Christensen (1979): "Modelling the Structure of Cost and Production for Multiproduct Firms," *Southern Economic Journal*, 46(1), 256–273.

Caves, D. W., L. R. Christensen, and M. W. Tretheway (1989): "Flexible Cost Functions for Multiproduct Firms," *Review of Economics and Statistics*, 62(3), 477–481.

Chamberlain, G. (1987): "Asymptotic efficiency in estimation with conditional moment restrictions," *Journal of Econometrics*, 34(3), 305–334.

Cherchye, L., T. Demuynck, B. D. Rock, and K. D. Witte (2011): "Nonparametric analysis of multi-output production with joint inputs," Center for Economic Studies - Discussion Papers ces11.35, Katholieke Universiteit Leuven, Centrum voor Economische Studiën.

Cherchye, L., B. D. Rock, B. Dierynck, F. Roodhoft, and J. Sabbe (2011): "Opening the 'black box' of efficiency measurement: input allocation in multi-output settings," Center for Economic Studies - Discussion papers ces11.10, Katholieke Universiteit Leuven, Centrum voor Economische Studiën.

De Loecker, J., P. K. Goldberg, A. K. Khandelwal, and N. Pavcnik (2012): "Prices, Markups, and Trade Reform," NBER Working Papers 17925, National Bureau of Economic Research, Inc.

De Rock, B., L. Cherchye, and F. Vermeulen (2008): "Analyzing cost efficient production behavior under economies of scope: a nonparametric methodology," ULB Institutional Repositary 2013/7536, ULB – Universite Libre de Bruxelles.

Dhyne, E., A. Petrin, and F. Warzynski (2013): "Prices, Markups and Quality at the Firm-Product Level," .

Doms, M., and E. J. Bartelsman (2000): "Understanding Productivity: Lessons from Longitudinal Microdata," *Journal of Economic Literature*, 38(3), 569–594.

Doraszelski, U., and J. Jaumandreu (2013): "RD and productivity: Estimating endogenous productivity," *Review of Economic Studies*, (80), 1338–1383.

Draganska, M., M. Mazzeo, and K. Seim (2009): "Beyond plain vanilla: Modeling joint product assortment and pricing decisions," *Quantitative Marketing and Economics*, 7(2), 105–146.

Dunne, T., S. Klimek, and J. James Schmitz (2010): "Competition and Productivity: Evidence from the Post WWII U.S. Cement Industry," Working Paper 10-29, Center for Economic Studies, U.S. Census Bureau.

Eaton, B. C., and N. Schmitt (1994): "Flexible Manufacturing and Market Structure," *American Economic Review*, 84(4), 875–888.

Eckel, C., L. Iacovone, B. Javorcik, and J. P. Neary (2011): "Multi-Product Firms at Home and Away: Cost- versus Quality-based Competence," CEPR. Discussion Papers 8186, Centre for Economic Policy Research.

Eckel, C., and J. P. Neary (2010): "Multi-Product Firms and Flexible Manufacturing in the Global Economy," *Review of Economic Studies*, 77(1), 188–217.

Feenstra, R., and H. Ma (2007): "Optimal Choice of Product Scope for Multiproduct Firms under Monopolistic Competition," NBER Working Papers 13703, National Bureau of Economic Research.

Gandhi, A., S. Navarro, and D. Rivers (2013): "On the Identification of Production Functions: How Heterogeneous is Productivity?," .

Goldberg, P. K., A. K. Khandelwal, N. Pavcnik, and P. Topalova (2010): "Multi-product Firms and Product Turnover in the Developing World: Evidence from India," *Review of Economics and Statistics*, 92(4), 1042–1049.

Harrison, A. E. (1994): "Productivity, imperfect competition and trade reform: Theory and evidence," *Journal of International Economics*, 36(1-2), 53–73.

Hill, R. J. (2004): "Constructing Price Indexes across Space and Time: The Case of the European Union," *American Economic Review*, 94(5), 1379–1410.

Hsieh, C.-T., and P. J. Klenow (2009): "Misallocation and Manufacturing TFP in China and India," *Quarterly Journal of Economics*, 124(4), 1403–1448.

Johnson, J. P., and D. P. Myatt (2003): "Multiproduct Quality Competition: Fighting Brands and Product Line Pruning," *American Economic Review*, 93(3), 748–774.

——— (2006): "Multiproduct Cournot Oligopoly," *RAND Journal of Economics*, 37(3), 583–601.

Ju, J. (2003): "Oligopolistic Competition, Technology Innovation, and Multiproduct Firms," *Review of International Economics*, 11(2), 346–359.

Klette, T. J., and Z. Griliches (1996): "The Inconsistency of Common Scale Estimators When Output Prices Are Unobserved and Endogenous," *Journal of Applied Econometrics*, 11(4), 343–361.

Levinsohn, J., and A. Petrin (2003): "Estimating Production Functions Using Inputs to Control for Unobservables," *Review of Economic Studies*, 70(2), 317–341.

Ma, H. (2009): "Firm Sorting, Ownership and Product Scope: Evidence from Chinese Enterprises," .

Marschak, J., and J. William H. Andrews (1944): "Random Simultaneous Equations and the Theory of Production," *Econometrica*, 12(3-4), 143–205.

Mayer, T., M. Melitz, and G. I. P. Ottaviano (2014): "Market Size, Competition, and the Product Mix of Exporters," *American Economic Review*, 104(2), 495–536.

Mundlak, Y. (1961): "Empirical Production Function Free of Management Bias," *Journal of Farm Economics*, 43(1), 44–56.

Neary, P. J. (2004): "Rationalizing the Penn World Table: True Multilateral Indices for International Comparisons of Real Income," *American Economic Review*, 94(5), 1411–1428.

Nocke, V., and S. R. Yeaple (2013): "Globalization and Multiproduct Firms," NBER Working Papers 9037, National Bureau of Economic Research, Inc.

OECD (2013): "OECD Indicators of Employment Protection," http://www.oecd.org.

Olley, G. S., and A. Pakes (1996): "The Dynamics of Productivity in the Telecommunications Equipment Industry," *Econometrica*, 64(6), 1263–1297.

Ornaghi, C., and I. V. Beveren (2011): "Using proxy variables to control for unobservable when estimating productivity: A sensitivity analysis," Discussion Papers (IRES - Institut de Recherches Economiques et Sociales) 2011029, Institut de Recherches Économiques et Sociales de l'Université Catholique de Louvain.

Panzar, J. (1989): *Technological determinants of firm and industry structure*vol. 1 of *Handbook of Industrial Organization*, chap. 1, pp. 3–59. Elsevier, 1 edn.

Raval, D. (2012): "Non Neutral Technology and the Microeconomic Production Function," .

Reynaert, M., and F. Verboven (2014): "Improving the performance of random coefficients demand models: The role of optimal instruments," *Journal of Econometrics*, 179(1), 83–98.

Roson, R. (2012): "Competition between Multiproduct Firms with Heterogeneous Costs," Working Papers $2012_14, Department of Economics, University of Venice"Ca'Foscari"$.

Schmitz, James A., J. (2005): "What Determines Productivity? Lessons from the Dramatic Recovery of the U.S. and Canadian Iron Ore Industries Following Their Early 1980s Crisis," *Journal of Political Economy*, 113(3), 582–625.

Seim, K. (2006): "An empirical model of firm entry with endogenous product-type choices," *RAND Journal of Economics*, 37(3), 619–640.

Syverson, C. (2004a): "Market Structure and Productivity: A Concrete Example," *Journal of Political Economy*, 112(6), 1181–1222.

——— (2004b): "Product Substitutability and Productivity Dispersion," *Review of Economics and Statistics*, 86(2), 534–550.

——— (2011): "What Determines Productivity?," *Journal of Economic Literature*, 49(2), 326–365.

Valmari, N. (2014): "Misspecified Production Functions: Product- vs. Firm-level Technologies," .

Wedervang, F. (1965): *Development of a population of industrial firms: the structure of manufacturing industries in Norway, 1930-1948.*, Scandinavian university books. Universitetsforlaget.

Wooldridge, J. (2009): "On estimating firm-level production functions using proxy variables to control for unobservables," *Economics Letters*, 104(3), 112–114.

Wooldridge, J. M. (2002): *Econometric Analysis for Cross Section and Panel Data*. MIT Press.

9 Tables and Figures

Table 1

PRODCOM	Title
16.10.10.33	Coniferous wood; sawn or chipped lengthwise, sliced or peeled, of a thickness > 6 mm, end-jointed, sanded or planed
16.10.10.33.10	Spruce wood (Picea abies Karst.), sanded or planed, end-jointed, sawn or chipped lengthwise, sliced or peeled, of a thickness > 6 mm
16.10.10.33.20	Pine wood (Pinus sylvestris L.), sanded or planed, end-jointed, sawn or chipped lengthwise, sliced or peeled, of a thickness > 6 mm
16.10.10.35	Spruce wood (Picea abies Karst.), fir wood (Abies alba Mill.)
16.10.10.37	Pine wood (Pinus sylvestris L.)
16.10.10.50	Wood, sawn or chipped lengthwise, sliced or peeled, of a thickness > 6mm (excluding coniferous and tropical woods and oak blocks, strips and friezes)
16.10.21.10	Coniferous wood continuously shaped (including strips and friezes for parquet flooring, not assembled)
16.10.23.03	Coniferous wood in chips or particles
16.10.23.05	Non-coniferous wood in chips or particles
16.10.41.00.10	Sawdust
16.10.41.00.20	Woodchips
16.10.41.00.40	Lathes, borders, etc.
16.10.41.00.60	Bark
16.10.41.00.80	Other wood waste (excluding sawdust, woodchips, bark, lathes, borders, pellets, briquettes etc.)
16.21.11.00	Plywood, veneered panels and similar laminated wood, of bamboo
16.21.12.14	Plywood consisting solely of sheets of wood (excluding of bamboo), each ply not exceeding 6 mm thickness, with at least one outer ply of non-coniferous wood (excluding trpical wood)
16.21.12.17	Plywood consisting solely of sheets of wood (excluding of bamboo), each ply not exceeding 6 mm thickness (excluding products with at least one outer ply of tropical wood or non-coniferous wood)
16.21.13.13	Particle board, of wood
16.21.21.18.30	Veneer for plywood, cross-banded plywood and other wood, of coniferous wood, sawn lengthwise, sliced or peeled, of a thickness <=6mm (excluding end-jointed, planed, sanded and board for manufacturing pencils)
16.21.21.18.80	Veneer for plywood, cross-banded plywood and other wood, of hardwood, sawn lengthwise, sliced or peeled, of a thickness <=6mm (excluding end-jointed, planed, sanded and board for manufacturing pencils)
16.21.22.00	Densified wood, in blocks, plates, strips or profile shapes

16.22.10.60	Parquet panels of wood (excluding those for mosaic floors)
16.23.11.10	Windows, French-windows and their frames, of wood
16.23.11.50	Doors and their frames and thresholds, of wood
16.23.19.00.12	Carpenter's produce for walls, of wood
16.23.19.00.16	Carpenter's produce for stairs, of wood
16.23.19.00.26	Components for sauna, of wood
16.23.19.00.32	Panel elements (also glulam and cellular panels), of wood
16.23.19.00.36	Ceiling elements, of wood
16.23.19.00.42	Glulam beams and columns
16.23.19.00.46	Vertical and horizontal beams (excluding glulam beams and columns)
16.23.19.00.52	Log frames for buildings of wood
16.23.19.00.90	Other carpenter's produce, of wood (excluding doors, windows, produce for floors, walls, stairs and sauna, panel and ceiling elements, beams, columns and log frames)
16.23.20.00.20	Residential buildings of wood, for permanent habitation
16.23.20.00.40	Residential buildings of wood, for recreational use
16.23.20.00.60	Saunas of wood (outdoor saunas, assembled or prefabricated)
16.23.20.00.90	Buildings of wood (assembled or prefabricated) (excluding residential buildings and saunas)
16.24.11.35	Box pallets and load boards of wood (excluding flat pallets)
16.24.13.20	Cases, boxes, crates, drums and similar packings of wood (excluding cable drums)
16.24.13.50	Cable-drums of wood
16.29.14.90	Other articles of wood (excluding pallet collars)

Table 2: Parameter estimates
PRODCOM 161: Sawmilling and planing of wood
PRODCOM 162: Manufacture of products of wood, cork, straw and plaiting materials

	Parameter estimate (standard error)
PRODCOM 161	
Materials	0.37 0.008
Labor	0.36 0.011
Capital	0.20 0.008
Price elasticity of demand	-1.29 0.020
PRODCOM 162	
Materials	0.73 0.002
Labor	0.13 0.003
Capital	0.18 0.003
Price elasticity of demand	-1.13 0.004
Prob[Chi-sq.(264)>J]	0.4632
Number of obs.	2053

Essay 2

Misspecified Production Functions: Product- vs. Firm-level Technologies

Unpublished

Misspecified Production Functions: Product- vs. Firm-level Technologies*

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Abstract

This paper describes the biases that arise when production functions are estimated under the standard assumption of a firm-level technology, while the true technologies are product-specific. The assumption of a firm-level technology implies that the technology parameters are identical across the various goods produced in the industry, and that a multiproduct firm produces all of its output with a single technology. To examine the implications of these simplifying assumptions, I estimate a firm-level production function on a dataset generated of an industry where two types of goods are produced with product-specific Cobb-Douglas production functions. I find that the biases in the estimated firm-level parameters are substantial even when the true product-specific technologies are very similar. The directions and the magnitudes of the biases are determined by intricate functions of the true product-specific technologies and the product scopes of the firms in the industry. The estimated productivity levels have a relatively low correlation with the true firm-level productivity levels when the firms' product scopes are heterogeneous, as they usually are.

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1 Introduction

Most industries[1] comprise dozens if not hundreds of products titles. These products are manufactured by firms with various product assortments. In the Finnish manufacturing sector, for example, plants' product scopes[2] vary from one to 82 titles, with more than 60% of the plants producing at least two titles.[3] Moreover, the plants' output assortments vary both in the product titles as well as the titles' output shares, both in time within plants, and across plants. Similar observations have been made also about the US manufacturing sector (Bernard, Redding and Schott, 2010). The empirical literature suggests that multiproduct firms have intra-firm productivity differences across product lines (Bernard, Redding and Schott, 2009, 2011; Mayer, Melitz and Ottaviano, 2014), and that changes in product assortments are important strategic choices (Eaton and Schmitt, 1994; Eckel, Iacovone, Javorcik, Neary, 2011; Johnson and Myatt, 2003, 2006; Ju, 2003; Roson, 2012).

Despite the aforementioned empirical facts, almost all production function estimates are based on the following two assumptions:[4] (1) technology parameters, i.e., output elasiticites of various inputs, are constant for all goods in the industry, and (2) a firm produces all of its titles with a single firm-level technology and productivity level. This paper describes the estimation biases that arise if these two assumptions of a firm-level technology are imposed when the true technologies are product-specific. The biases are characterized by estimating a firm-level production function on various simulated datasets

[1] Industries are typically defined at the two-digit level. The European PRODCOM classification for the manufacturing sector, for example, at the two-digit level is 10 Manufacture of Food Products, 11 Manufacture of Beverages, 12 Manufacture of Tobacco Products, 13 Manufacture of Textiles, 14 Manufacture of Wearing Apparel, 15 Manufacture of Leather and Related Products, 16 Manufacture of Wood and Products of Wood and Cork, except Furniture, Manufacture of Articles of Straw and Plaiting Materials, 17 Manufacture of Paper and Paper Products, 18 Printing and Reproduction of Recorded Media, 19 Manufacture of Coke and Refined Petroleum Products, 20 Manufacture of Chemicals and Chemical Products, 21 Manufacture of Basic Pharmaceutical Products and Pharmaceutical Preparations, 22 Manufacture of Rubber and Plastic Products, 23 Manufacture of Other Non-Metallic Mineral Products, 24 Manufacture of Basic Metals, 25 Manufacture of Fabricated Metal Products, except Machinery and Equipment, 26 Manufacture of Computer, Electronic and Optical Products, 27 Manufacture of Electrical Equipment, 28 Manufacture of Machinery and Equipment n.e.c., 29 Manufacture of Motor Vehicles, 30 Manufacture of Other Transport Equipment, 31 Manufacture of Furniture, 32 Other Manufacturing, 33 Repair and Installation of Machinery and Equipment.

[2] Multiproduct firms exist due to economies of scope. See, for example, Panzar, 1989 for how production technology affects firm and industry structure.

[3] According to the Industrial output data of Statistics Finland on years 2004 - 2011.

[4] For an exception, see De Loecker, Goldberg, Khandelwal and Pavcnik, 2012. There is also an early literature on estimating cost functions of multiproduct firms. See, for example, Brown, Caves and Christensen, 1979, and Caves, Christensen and Tretheway, 1980. The early multiproduct cost functions allow for the fact that production technologies across goods vary, but they do not correct the typical endogeneity problems such as the simultaneity or selection bias.

where the true production technologies are product-specific.

The following example illustrates the two assumptions. Consider an industry called "Manufacture of Wood and of Products of Wood and Cork", which constitutes division 16 of the European PRODCOM classification. This industry comprises titles such as Sawmilling and Planing of Wood (16.10) and Prefabricated Wooden Buildings (16.23.20). Without any information about the production technologies used - like an econometrician often is when estimating firms' production functions - it is impossibe to tell whether the goods have been produced with similar technologies or not. I can only make an educated guess that sawmilling and planing of wood is likely to involve less processing than production of wooden buildings, and as a result the production technologies for the two titles may be different. More precisely, the output elasticities, for example, of materials, labor, and capital, are likely to differ across the production technologies for the two goods.

Moreover, a firm that manufactures both sawmilled or planed wood and wooden buildings may not be equally productive in manufacturing both titles. This relates to the concept of core competency that has been discussed in the context of multiproduct firms (for exampe Bernard, Redding and Schott, 2011). In the production function literature core competency refers to a circumstance where a firm is more productive in manufacturing some goods than others. In addition, the firm may have increasing or decreasing returns to scale in the production of, say, wooden buildings, but these returns to scale may not spill over to the production of sawmilled or planed wood, for example. This would imply that the production technologies are product-, not firm-level functions.

In this paper I consider the implications of misspecifying production functions as firm-level instead of product-level functions. I simulate a dataset where two types of goods are produced with product-level Cobb-Douglas technologies. I use the dataset to estimate a firm-level production function, assuming away the existence of product-level production technologies, like in the literature. Estimations on the simulated datasets show that the biases in the estimated firm-level parameters are substantial even when the true product-level technologies are very similar. The directions and the magnitudes of the parameter biases are intricate functions of the true product-level technologies and the product scopes of the firms in the industry. Also the residuals, which are often considered to be estimates of the unobservable productivity levels, are affected: the more heterogeneous the product scopes of the firms, the lower the correlation between the estimated and true firm-level productivity levels.

In the next section I relate this study to the relevant literature. In section 3 I explain how the simulations are carried out. The results are presented in section 4. Section 5 concludes.

2 Literature

Estimation of production functions is subject to various identification challenges. The most well-known problems are simultaneity and selection biases, collinear variables, unobservable variables such as price and quality of inputs and outputs, and functional form misspecifications (see, for example, Ackerberg, Benkard, Berry and Pakes, 2007). They are discussed in the literature review of Valmari (2014), where I propose an estimation strategy for product-level production functions. In this paper I relate my study primarily to the literature on aggregation of production functions, which has evolved within the macro literature but has not gained attention among microeconomists who estimate firms' production functions.

A key element in the neoclassical macroeconomics literature is the aggregate production function. It is constantly estimated despite numerous critical remarks that the aggregate production function does not have a sound theoretical foundation (Felipe and Fisher, 2003). There are two types of issues related to the aggregation of production functions: aggregation over various inputs and outputs, and aggregation over firms when not all inputs are efficiently allocated. Felipe and Fisher discuss the theoretical literature on the aggregation problem.

Klein (1946a, 1946b) initiated the literature on production function aggregation. His objective was to write an aggregate production function as a purely technological relationship, independent of behavioral assumptions such as profit maximization. However May (1946) pointed out that even the micro production functions assume optimization. Pu (1946) noted that if the macro variables are not derived from micro variables that satisfy equilibrium conditions, neither will the macroeconomic equilibrium conditions hold.

The first major findings were made by Leontief and Nataf. Leontief (1947a, 1947b) provides necessary and sufficient conditions for aggregation of variables into homogeneous groups within a firm. Aggregation is possible if and only if the marginal rates of substitution among variables in the aggregate are independent of the variables outside of it. This assumption may hold for some real-life producers but it is unlikely to hold for all of them. Nataf (1948) considers aggregation over different production functions. He finds that aggregation over different functions is possible if and only if the micro production functions are additively separable in capital and labor.

Fisher (1969, 1993) notes that without imposing an efficiency condition, an aggregate function almost never exists. He provides conditions for the existence of aggregates of capital, labor and output under some presumptions. Fisher assumes that, first, labor is allocated across firms efficiently, second, capital is firm-specific and hence capital markets do not exist, and third, firm-level production functions have constant returns to scale. Even under these strong assumptions the conditions for the existence of aggregate production functions are stringent. The aggregates exist only if, first, firm-level production functions are identical except for the capital efficiency coefficient, second, all firms employ different types of labor in the same proportion, i.e., specialization in labor is ruled out, and third, all firms produce all goods in the same proportions, i.e., specialization in output is ruled out. Felipe and Fisher conclude that the conditions under which a well-behaved aggregate production function can be derived are so stringent that actual economies are unlikely to satisfy them.

The firm-level aggregation problem I look at has similarities with the macroeconomic counterpart, albeit the problems are not identical. In the case of firm-level data, aggregation takes place over multiple inputs and outputs, and over various production functions, but in contrast to the macroeconomic literature, decision-makers are not aggregated over. I am not aware of a study that looks at the implications of aggregation to the firm-level. The findings of this study are therefore relevant not only for economists estimating firms' production functions, but also to the macro literature which is based on the assumption that firm-level production functions are well-behaved (Felipe and Fisher, 2003).

Another aspect of aggregation has been recently raised in the micro-level production function literature, however. Bernard, Redding and Schott (2009) note that most firms make their production decisions at a more disaggregated level than observed in the data, and therefore studied in the productivity literature. They argue that, in addition to the functional form misspecification of assuming homogenous production functions across goods, firms' product choices and productivity levels are correlated, like in the traditional selection problem. Bernard, Redding and Schott write a theoretical model of industry equilibrium where firms choose one out of two heterogeneous goods. Production technologies for the goods vary such that one of the goods involves a lower variable and a higher fixed cost than the other. The outcome of the model is that high productivity firms, defined as firms whose productivity exceed a certain threshold, produce the goods with a low variable and a high fixed cost. Because the high productivity firms can produce their output even at a lower variable cost than the low productivity firms, they can cover the high fixed cost by selling a large output quantity at a low price. The low productivity firms manufacture the goods with a high variable cost and a low fixed cost, respectively. Bernard, Redding and Schott note that in production function estimation, variation in firm productivity cannot be distinguished from variation in the production technologies, the two of which are correlated due to the endogenous product choices, which leads to a product bias in productivity measurement. They find that whether the inter-firm differences in measured productivity are greater or smaller than the true productivity differences depends on the divergence between the variable and fixed cost parameters. If the difference in the variable cost parameters is large relative to the difference in the fixed cost parameters, the measured productivity differences are larger than the true ones. Bernard, Redding and Schott assume that productivity is a firm-level variable, and hence sorting according to product-specific productivity is ruled out.

The paper of Bernard, Redding and Schott and this study are are both based on the observation that production technologies may differ across products even within industries. However the production function estimation biases considered in these studies are different both in their causes and their implications. Bernard, Redding and Schott consider the bias in measured productivity caused by by ignoring endogenous product selection. This study, in contrast, looks at the functional form misspecification problem due to assuming away product-level technologies, which has implications on the estimated technology parameters as well as the measured productivity levels.

3 Simulations

If production technologies are assumed to be firm-level functions while they actually are product-specific, the estimation equations are misspecified. First, the output elasticities of the inputs may not be equal across product lines. Second, if multiproduct firms are present, productivity levels are not necessarily constant across product lines within firms. Third, even if all product-level production functions were identical, they would not add up to a firm-level function without changing the functional form, unless the returns to scale were constant for all the technologies.

To find how production function estimates are determined under the above functional form misspecfication, I run simulations. I first generate a dataset where the product-level technologies are known. I then estimate the production functions at the firm-level, as is the practice in the empirical literature, and compare the firm-level estimates to the true product-level technologies.

I consider functional form misspecification for the Cobb-Douglas technology. Back in 1955, Houthakker characterized the Cobb-Douglas function as sufficiently consistent with notions of economic theory to be an useful approximative device, even though the function is not firmly established as an empirical regularity. Many microeconomists still agree with Houthakker, as even today the Cobb-Douglas function dominates the literature on firms' production. For example, most structural estimation strategies assume the Cobb-Douglas technology (e.g. Olley and Pakes, 1996; Levinsohn and Petrin, 2003; Ackerberg, Caves and Frazer, 2006; and De Loecker, Goldberg, Khandelwal and Pavcnik, 2012). The only exception is the translog approximation used by Gandhi, Navarro and Rivers (2013). Hence the findings of this study cater to interpreting a large number of empirical papers.

3.1 Data generation

The data generating process for the simulations is such that the implications of the functional form misspecification for production function estimates are transparent. The data generating process is also the simplest one that allows for substitution between inputs and output types within firms. Any identification issues, such as simultaneity, selection, and collinearity problems, unobservable price and quality differences, technological change, and small sample size, are assumed away.

I generate datasets where firms produce one or two goods, each with the respective production technology. Choices on what product types to produce are exogenous.[5] In reality the number of goods produced and technologies used in an industry may of course be more than two. However the qualitative effects of the misspecification are likely to be the same when the number of true technologies is greater.

The data generated is an outcome of firms maximizing static profits. Firms are typically assumed to have at least one dynamic factor of production, capital and perhaps also knowledge investments. Decisions on dynamic factors affect production and profit also after the current period. Whether an input is a static variable or has dynamic implications does not change the effects of the functional form misspecification studied in this paper, however. Therefore, to simplify the data generating process, the firms have two static inputs, labor L_{ij} and capital K_{ij}:

$$Q_{ij} = L_{ij}^{\beta_{Li}} K_{ij}^{\beta_{Ki}} \exp\left(\omega_{ij}\right). \tag{1}$$

The output elasticities of the two inputs, β_{Li} and β_{Ki}, are product-specific. Total factor productivity, which is product- and firm-specific, is denoted by $\exp\left(\omega_{ij}\right)$. I treat L_{ij} and K_{ij} as exogenous to the unobservable productivity level ω_{ij} both in the data generation as well as in the estimation process. Allowing L_{ij} and K_{ij} to be endogenous to ω_{ij} would require the consequent endogeneity bias to be treated by using an appropriate estimator. However, I don't know how the present estimators, such as Olley and Pakes (1996), Levinsohn and Petrin (2004), Ackerberg, Caves and Frazer (2006), or Wooldridge (2009), perform if the assumption of firm-level production functions does not hold. Allowing for endogeneity would make the analysis less straightforward because the effects of the functional form misspecification would have to be distinguished from the misperformance of the estimator in the presence of the functional form misspecification. Hence I assume L_{ij} and K_{ij} to be exogenous to ω_{ij}.[6]

[5] As discussed in the literature review, Bernard, Redding and Schott (2009) discuss the implications of ignoring endogenous product choices in production function estimation.

[6] Because both inputs, L_{ij} and K_{ij}, are static decision variables and exogenous to the productivity level ω_{ij}, a cross-sectional dataset is sufficient in this study. In contrast, if at least one of the inputs were dynamic, generation of these inputs would produce a longitudinal dataset. If at least one of the inputs were endogenous to ω_{ij}, the estimation methods of Olley and Pakes (1996), Levinsohn and Petrin (2004), and Ackerberg, Caves and Frazer (2006), for example, could be used for identifying the production function, but that would require a longitudinal dataset of at least two consecutive time periods.

The firm chooses its input and hence also output levels as a function input and output prices. Input prices, W_j for L_{ij}, and R_j for K_{ij}, vary across firms. In the output market firms face downward-sloping demand curves with a product-firm -specific demand level:

$$Q_{ij} = \exp\left(\alpha_{ij}\right) P_{ij}^{\eta_i}, \tag{2}$$

where P_{ij} is price of good i produced by firm j, η_i is price elasticity of demand for good i, and α_{ij} captures the good-firm -specific demand level. Variation in output demand induces firms to substitute between goods, while variation in W_j and R_j induce substitution between the two inputs.

The firm sets inputs L_{ij} and K_{ij} to maximize the static profits in all the product lines i it is active in:

$$\max_{L_{ij},K_{ij}} \Pi_{ij} = \sum_i P_{ij}Q_{ij} - W_j L_{ij} - R_j K_{ij} \tag{3}$$

Substituting in the inverse demand functions, $P_{ij} = \left(Q_{ij}\left(\exp(\alpha_{ij})\right)^{-1}\right)^{\frac{1}{\eta_{ij}}}$, and the production functions, the static profit maximization problem becomes:

$$\max_{L_{ij},K_{ij}} \Pi_{ij} = \sum_i \left(\exp(\alpha_{ij}+\varepsilon_{ij})\right)^{-\frac{1}{\eta_i}} \left(L_{ij}^{\beta_{Li}} K_{ij}^{\beta_{Ki}} \exp\left(\omega_{ij}\right)\right)^{\frac{1}{\eta_i}+1} - W_j L_{ij} - R_j K_{ij} \tag{4}$$

The first-order conditions for static profit maximization for firm j producing product i are:

$$\begin{aligned}
\frac{\partial Lagr}{\partial L_{ij}} &= \left(\frac{1}{\eta_i}+1\right)\left(\exp(\alpha_{ij})\right)^{-\frac{1}{\eta_i}}\left(L_{ij}^{\beta_{Li}}K_{ij}^{\beta_{Ki}}\exp\left(\omega_{ij}\right)\right)^{\frac{1}{\eta_i}+1}\frac{\beta_{Li}}{L_{ij}} - W_j = 0 && (5)\\
\frac{\partial Lagr}{\partial K_{ij}} &= \left(\frac{1}{\eta_i}+1\right)\left(\exp(\alpha_{ij})\right)^{-\frac{1}{\eta_i}}\left(L_{ij}^{\beta_{Li}}K_{ij}^{\beta_{Ki}}\exp\left(\omega_{ij}\right)\right)^{\frac{1}{\eta_i}+1}\frac{\beta_{Ki}}{K_{ij}} - R_j = 0 && (6)\\
\forall\ i &= [1, n_j]\\
\forall\ j &= [1, J]
\end{aligned}$$

These first-order conditions give the profit-maximizing inputs L_{ij}^* and K_{ij}^*.

3.1.1 The number of goods produced

I generate datasets for four scenarios: (1) 1/2 of the firms produce good 1, and the other 1/2 of the firms produce good 2, (2) 1/3 of the firms produce good 1, another 1/3 of the firms produce good 2, and the remaining 1/3 of the firms produce both goods, (3) 1/10 of the firms produce good 1, another 1/10 of the firms produce good 2, and the remaining 8/10 of the firms produce both goods, and (4) all firms produce both goods. Demand for the goods is not correlated within firms, nor are the product-specific productivity levels. The only difference between the four datasets is the exogeneous variation in product selection.

In the first scenario all firms are single-product firms. According to datasets on firms in the manufacturing sector, such a scenario is very unlikely (Bernard, Redding and Schott, 2010, Valmari, 2014), but because most studies implicitly assume single-product firms, results for the scenario may also be of interest. The other three cases are empirically more relevant. In the US manufacturing sector 40% of the firms produce at least two goods (Bernard, Redding and Schott, 2010), while more than 60% of Finnish manufacturing plants produce multiple goods.[7] The two scenarios with 1/3 and 8/10 of the firms producing two goods may therefore be considered as illustrations of a national manufacturing industry, for example. The fourth case where all firms are multiproduct producers corresponds to a dataset on exporting firms, where virtually all firms are multiproduct firms (Bernard, Jensen, Redding and Schott, 2007).

3.1.2 Production function parameters

To cover different production technology combinations that may prevail within industries, I consider altogether 18 different sets of product-level technologies, displayed in Table 1. The 18 cases differ in the technology parameters: in the technologies' output elasticities and returns to scale. Apart from the technology parameters, the data generating process for the 18 cases is identical.

In cases 1 to 9 (10 to 18), the technologies have equal (unequal) returns to scale. In cases 1 to 3 (4 to 6) [7 to 9], both technologies have constant returns to scale, $\beta_{Li}+\beta_{Ki}=1$, (increasing returns to scale, $\beta_{Li}+\beta_{Ki}>1$) [decreasing returns to scale, $\beta_{Li}+\beta_{Ki}<1$]. In cases 10 to 15, technology for good 1 has constant returns to scale, while technology for good 2 has increasing (cases 10 to 12) or decreasing (cases 13 to 15) returns to scale. In cases 16 to 18, technology for good 1 has increasing returns to scale, and technology for good 2 has decreasing returns.

[7] According to the Industrial output data of Statistics Finland on years 2004 - 2011.

In all cases, technology for good 1 has higher output elasticity for L than for K, $\beta_{L1} > \beta_{K1}$. Depending on the case, β_{L1} ranges between 0.71 and 0.69, while β_{K1} is 0.3 across all the cases. The output elasticities of the technology for good 2 can be divided into three groups. In the first group, the elasticities are identical (cases 1, 4, 7 with equal returns to scale), or very close to the parameters of the technology for good 1 (cases 10, 13, 16 with unequal returns to scale). In the second group, the two elasticities of the technology for good 2 are exactly or approximately 0.5 each (cases 2, 5, 8, 11, 14 and 17), and hence the parameters differ from those of technology for good 1 by about 0.2 in absolute value. In the third group, technology for good 2 has lower output elasticity for L than for K, such that β_{L1} is close to β_{K2}, and β_{K1} is close to β_{L2}.

3.1.3 Other exogeneous parameters and variables

There are four exogenous parameters or variables that induce firms to substitute between the inputs and, in the case of two-good producers, between the output and the respective technology types. These exogenous parameters and variables yield identifying variation in the input choices for the two goods. Factor prices W_j and R_j induce substitution between the inputs. They are normally distributed with mean 10 and standard deviation 1. To avoid the problem of collinear inputs, the input prices are not correlated. Demand for the goods, i.e., the price elasticity of demand η_i and the level of demand α_{ij}, bring about variation in the two output types. Demand for both types of goods is elastic with price elasticity 1.05,[8] while α_{ij} is normally distributed with mean 23 and standard deviation 0.1.

3.2 Estimation

Due to how the input and output data has been generated, the product-level production functions may be estimated by OLS to obtain unbiased and efficient estimates. To examine the estimates obtained when imposing the assumption of a firm-level technology, I estimate the following equation:

$$Q_j = L_j^{\beta_L} K_j^{\beta_K} \exp(\omega_j). \tag{7}$$

[8] If demand was inelastic, the model would imply negative input choices.

where the dependent variable is $Q_j = \sum_{i=1}^{N_j} Q_i$, and the explanatory variables are $L_j = \sum_{i=1}^{N_j} L_{ij}$ and $K_j = \sum_{i=1}^{N_j} K_{ij}$. After taking logarithms the equation can be estimated by OLS.

Before turning to the estimation results, I consider how the above estimation equation compares with the true firm-level production function aggregates when all the producers are one-product firms (scenario 1), and when at least one of the firms is a multiproduct firm (scenarios 2 - 4).

3.2.1 One-product firms (scenario 1)

Consider product-level production functions for N goods, denoted by subscript i. All the production technologies use H types of inputs. The output elasticities of the inputs are captured in a product-specific parameter vector β_i, which may vary across goods i. Taking logs, the produduction functions of the firms are written in matrix form as follows:

$$\mathbf{q}_i = \mathbf{X}_i \boldsymbol{\beta}_i + \boldsymbol{\omega}_i \tag{8}$$

where q_i is the log of output, X_i is the log of inputs, and residual ω_i is the log of productivity level for goods of type i.

The standard estimation strategy of the literature, in this scenario where all firms are one-product firms, implies assuming that all production functions i are identical:

$$\mathbf{q} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\omega}. \tag{9}$$

Imposing the assumption of a single technology can be considered as an example restricted least squares estimation with the following parameter restriction:

$$\boldsymbol{\beta}_i = \boldsymbol{\beta}_g \ \forall \ i = [1, N], \ \forall \ g = [1, N]. \tag{10}$$

The restricted least squares estimator is (for example, Greene, 2002):

$$\widehat{\boldsymbol{\beta}}_R = \widehat{\boldsymbol{\beta}} - \left(\mathbf{X}'\mathbf{X}\right)^{-1} \mathbf{R}' \left[\mathbf{R}\left(\mathbf{X}'\mathbf{X}\right)^{-1} \mathbf{R}'\right]^{-1} \left(\mathbf{r} - \mathbf{R}\widehat{\boldsymbol{\beta}}\right) \tag{11}$$

where the parameter restriction is

$$\mathbf{R}\boldsymbol{\beta} = \mathbf{r}. \tag{12}$$

In the case of two-input technologies and two goods,[9] (10) translates into (12) such that:

$$\mathbf{R} = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix}, \tag{13}$$

$$\boldsymbol{\beta} = \begin{bmatrix} \boldsymbol{\beta}_1 \\ \boldsymbol{\beta}_2 \end{bmatrix}, \text{ and} \tag{14}$$

$$\mathbf{r} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}. \tag{15}$$

If the constraints hold in reality, the restricted estimator $\widehat{\boldsymbol{\beta}}_R$ equals the unrestricted $\widehat{\boldsymbol{\beta}}$. If the constraints are not correct, the estimators are different. In that case the restriced estimator $\widehat{\boldsymbol{\beta}}_R$ consists of the unrestricted estimator and a correction term that accounts for the failure of the unrestricted estimator to satisfy the constraints. In short, whenever the constraint is not true, the restricted estimator $\widehat{\boldsymbol{\beta}}_R$ is biased. The directions of such biases, as well as the implications on the residuals, are considered by the simulations.

3.2.2 N-product firms (scenarios 2, 3, and 4)

Like the majority of production functions, Cobb-Douglas is non-linear in inputs. As a consequence, even if the product-level production functions were identical, they would not necessarily add up to a firm-level function without changing the functional form and the parameters. Hence finding the correct level of specification is important whenever using a production function that is non-linear in inputs.

Assume now that all firms j produce N_j types of goods i using N_j separate product-level Cobb-Douglas production functions, again with H types of inputs for each good. The

[9] In a general case of H inputs and N goods, R is a matrix of size $(H(N-1), HN)$ where each odd (even) row has a 1 in the first (second) column, a -1 in each cell $(f, f+H)$ $\forall$ $f = [1, H(N-1)]$, and zeros elsewhere. β is a vector of length HN where the product-specific β_i's are stacked, and r is a vector of 0's of length HN.

product-level production function is (now writing in scalar form):

$$Q_{ij} = \prod_{h=1}^{H} X_{hij}^{\beta_{hi}} \exp(\omega_{ij}) \tag{16}$$

and hence the total output of the firm is given by:

$$\sum_{i=1}^{N_j} Q_{ij} = \sum_{i=1}^{N_j} \prod_{h=1}^{H} X_{hij}^{\beta_{hi}} \exp(\omega_{ij}) \, . \tag{17}$$

Instead of the true aggregate function above, the following equation is typically estimated in the literature:

$$\sum_{i=1}^{N_j} Q_{ij} = \prod_{h=1}^{H} \left(\sum_{i=1}^{N_j} X_{hij} \right)^{\widehat{\beta}_h} \exp(\widehat{\omega}_j) \, . \tag{18}$$

To see how (17) and (18) differ, I rewrite the true aggregate using the following auxiliary terms:

$$\overline{X}_{hj} = \frac{\sum_i^{N_j} X_{hij}}{N_j} \tag{19}$$

$$x_{hij} = 1 + \frac{X_{hij} - \overline{X}_{hj}}{\overline{X}_{hj}} \tag{20}$$

$$\Delta\beta_{hi} = \beta_{hi} - \widehat{\beta}_h \tag{21}$$

where $\widehat{\beta}_h$ is the input elasticity of output estimated for input type h. The true firm-level output aggregate can then be rewritten as:

$$\sum_{i}^{N_j} Q_{ij} = \sum_{i}^{N_j} \left(\prod_{h=1}^{H} \left(\overline{X}_{hj}^{\widehat{\beta}_h + \Delta\beta_{hi}} x_{hij}^{\widehat{\beta}_h + \Delta\beta_{hi}} \right) \exp\left(\omega_{ij}\right) \right) \tag{22}$$

$$\sum_{i}^{N_j} Q_{ij} = \prod_{h=1}^{H} \overline{X}_{hj}^{\widehat{\beta}_h} \sum_{i}^{N_j} \left(\prod_{h=1}^{H} \left(\overline{X}_{hj}^{\Delta\beta_{hi}} x_{hij}^{\widehat{\beta}_h + \Delta\beta_{hi}} \right) \exp\left(\omega_{ij}\right) \right) \tag{23}$$

$$\sum_{i}^{N_j} Q_{ij} = \prod_{h=1}^{H} \left(N_j \overline{X}_{hj}\right)^{\widehat{\beta}_h} \sum_{i}^{N_j} \left(\prod_{h=1}^{H} \left(\overline{X}_{hj}^{\Delta\beta_{hi}} x_{hij}^{\widehat{\beta}_h + \Delta\beta_{hi}} \right) \exp\left(\omega_{ij}\right) \right) \prod_{h=1}^{H} N_j^{-\widehat{\beta}_h} \tag{24}$$

$$\sum_{i}^{N_j} Q_{ij} = \underbrace{\prod_{h=1}^{H} \left(\sum_{i}^{N_j} X_{hij} \right)^{\widehat{\beta}_h}} \underbrace{\sum_{i}^{N_j} \left(\prod_{h=1}^{H} \left(X_{hij}^{\Delta\beta_{hi}} x_{hij}^{\widehat{\beta}_h} \right) \exp\left(\omega_{ij}\right) \right) N_j^{-\sum_{h=1}^{H} \widehat{\beta}_h}}. \tag{25}$$

Note that in the true aggregate rewritten in (25), the first term, $\prod_{h=1}^{H} \left(\sum_i^{N_j} X_{hij}\right)^{\widehat{\beta}_h}$, is equal to the deterministic part of the typical estimation equation in the literature (18). This implies that if the typical estimation equation (18) is adopted when the data generating process is product-specific (17), the estimated residual $\exp\left(\widehat{\omega}_j\right)$ is in fact the second part of (25). Clearly, the second part of (25) is not a term of unobservable productivity or output measurement error only. Instead, the firm-level productivity term estimated in the literature is a function of the true product-specific technology parameters β_{hi} and the product-level input allocations X_{hij}, as well as the true product-specific productivity levels ω_{ij}. In other words, the residual $\widehat{\omega}_j$ of the typical estimation equation (18) captures any output that remains unexplained by the deterministic part of the estimation equation (18). As a consequence, the distribution of $\widehat{\omega}_j$ may provide an unrealistic description of true productivity ω_{ij}.

As the logarithm of the estimation equation involves a logarithm of a sum, there is no analytical solution to how the parameter estimate $\widehat{\beta}_h$ is determined. The parameter estimates are therefore considered using simulations.

4 Results

The firm-level estimation equation (7) is misspecified when the true techologies are product-specific. Hence, a one-to-one comparison between the firm-level estimates and the true parameters cannot be made. Instead, I contrast the estimates with the two product-level technologies. I also compare the estimated and true returns to scale, as well as the estimated and the true firm-level productivity[10] terms. The estimation results are displayed in Tables 2, 3, 4 and 5 for the four different scenarios.

I start by characterizing the biases in the estimated firm-level parameters $\widehat{\beta}_L$ and $\widehat{\beta}_K$. Unbiased estimates are obtained only in exceptional cases. Only if the product-specific technologies are identical, and all firms produce the same number of goods (i.e., cases 1, 4 and 7 in scenarios 1 and 4, Tables 2 and 5), or the true technologies are not only identical but also subject to constant returns to scale (i.e., case 1 in scenarios 2 and 3, Tables 3 and 4), the firm-level estimates are unbiased. These circumstances are hardly realistic.

Consider first scenario 1 where all firms are single-product firms, one half of the firms producing good 1 and the other half producing good 2. Of the cases where both product-level technologies have constant returns to scale, cases 2 ($\beta_{L1} = 0.7$, $\beta_{K1} = 0.3$ and $\beta_{L2} = 0.5$, $\beta_{K2} = 0.5$), 5 ($\beta_{L1} = 0.71$, $\beta_{K1} = 0.3$ and $\beta_{L2} = 0.51$, $\beta_{K2} = 0.5$) and 8 ($\beta_{L1} = 0.69$, $\beta_{K1} = 0.3$ and $\beta_{L2} = 0.49$, $\beta_{K2} = 0.5$) stand out. The product-level technology parameters for a given input differ by 0.2 in absolute value, and the industry output shares of the two goods are no more different than 52% and 48%. Yet the estimated firm-level parameter estimates are identical (cases 2 and 5) or very close (case 8) to the parameters of the technology for good 1, and hence clearly biased from the parameters of the technology for good 2.

Estimates in cases 10 to 18 of scenario 1, where the returns to scale of the two product-level technologies differ, are subject to considerably higher parameter biases. In case 10 the true technologies are almost identical ($\beta_{L1} = 0.7$, $\beta_{K1} = 0.3$ and $\beta_{L2} = 0.71$, $\beta_{K2} = 0.3$) and the difference in the returns to scale is small (only 0.01), but the firm-level estimates have a substantial upward bias for labor ($\widehat{\beta}_L = 1.07$), and a downward bias for capital ($\widehat{\beta}_K = 0.10$). In case 11 the biases go in the opposite direction: $\widehat{\beta}_L$ is biased downwards ($\beta_{L1} = 0.7$, $\beta_{L2} = 0.51$ and $\widehat{\beta}_L = 0.34$), and $\widehat{\beta}_K$ is biased upwards ($\beta_{K1} = 0.3$, $\beta_{K2} = 0.5$ and $\widehat{\beta}_K = 0.68$). The firm-level estimates are not even between the true parameters in either of the cases. In cases 14 ($\beta_{L1} = 0.7$, $\beta_{K1} = 0.3$ and $\beta_{L2} = 0.49$, $\beta_{K2} = 0.5$), 16 ($\beta_{L1} = 0.71$, $\beta_{K1} = 0.3$ and $\beta_{L2} = 0.69$, $\beta_{K2} = 0.3$) and 17 ($\beta_{L1} = 0.71$, $\beta_{K1} = 0.3$ and

[10] The true firm-level productivity is computed by taking a weighted average of the product-specific productivity levels, where the weights are the output shares generated with the product-level inputs and productivity level $\exp(\omega_{ij}) = 1$ for each good: $\exp(\omega_j) = \frac{L_{1j}^{\beta_{L1}} K_{1j}^{\beta_{K1}}}{\sum_i L_{1j}^{\beta_{L1}} K_{1j}^{\beta_{K1}}} \exp(\omega_{1j}) + \frac{L_{2j}^{\beta_{L1}} K_{2j}^{\beta_{K1}}}{\sum_i L_{1j}^{\beta_{L1}} K_{1j}^{\beta_{K1}}} \exp(\omega_{2j})$

$\beta_{L2} = 0.49$, $\beta_{K2} = 0.5$), where the technology for product 2 is subject to decreasing returns to scale, $\widehat{\beta}_K$ is actually negative. Perhaps the most surprising estimates are obtained for case 16, where the two technologies are rather similar in the magnitudes of the output elasticities, with a difference in returns to scale of 0.02: $\widehat{\beta}_L$ is 2.06, a multiple of either of the true output elasticities of labor, and $\widehat{\beta}_K$ is -0.46, substantially below zero.

The estimates for the other three scenarios, where some or all firms produce multiple goods, are displayed in Tables 3 to 5. The parameter biases are similar in direction as in scenario 1 with single-product firms, but the magnitudes of the biases are somewhat lower. The greater the share of two-product firms, the smaller the biases. However even in scenario 3, where 80% of the firms produce two goods, the parameter biases are substantial. In cases 10 to 18, where the true technologies have slightly different returns to scale, none of the estimated firm-level technologies have parameters, $\widehat{\beta}_L$ and $\widehat{\beta}_K$, that both fall in between the true product-specific parameters, β_{L1} and β_{L2}, and β_{K1} and β_{K2}. Again even negative parameter estimates are obtained. The biases are lowest, albeit clearly different from zero, in scenario 4 where all firms produce two goods. For example, in case 17 the firm-level parameter estimates, $\widehat{\beta}_L = 0.80$ and $\widehat{\beta}_K = 0.21$, are clearly outside the ranges of the product-specific parameters, $\beta_{L1} = 0.71$, $\beta_{K1} = 0.3$ and $\beta_{L2} = 0.49$, $\beta_{K2} = 0.5$.

Two characteristics of the two true production technologies determine the directions of the parameter biases. First, when the true production technologies of the two goods are asymmetric in the sense that $|\beta_{Li} - \beta_{Ki}| > |\beta_{Lh} - \beta_{Kh}|$, where i stands for good 1 and h for good 2, or vice versa, then the parameter estimates are biased away from the true parameters of the technology for good h, the directions of the biases being towards the parameters of the technology for good i. Second, when the two true technologies have different returns to scale, i.e., $\beta_{Li} + \beta_{Ki} > \beta_{Lh} + \beta_{Kh}$, the parameter estimates are biased away from the true parameters of the technology for good h, the directions of the biases being towards the parameters of the technology for good i. When technology i is more asymmetric, i.e., $|\beta_{Li} - \beta_{Ki}| > |\beta_{Lh} - \beta_{Kh}|$, and technology h has higher returns to scale, $\beta_{Li}+\beta_{Ki} < \beta_{Lh}+\beta_{Kh}$, the biases in the parameter estimates are a mix of the two opposite effects. Depending on the parameters of the two true production technologies, and hence on the magnitudes of the estimation biases, the estimates may or may not be in between the parameters of the two true technologies. The sizes of the parameter bias grow in two characteristics. First, the greater the difference in the returns to scale between the two true technologies, the greater the bias. Second, the more there are one-good producers, the greater the bias.

When all firms are one-good firms (scenario 1) or all firms are two-good firms (scenario 4), and the true technologies have equal returns to scale, the estimated returns to scale are correctly estimated. If the true technologies have unequal returns to scale, the estimated returns are over- or underestimated in the case of one-good firms, and over-estimated in the case of two-good firms. When both one- and two-good firms are present (scenarios 2 and 3), the returns to scale are over- or underestimated depending on how similar the true technologies are. The more similar (different) the technologies, the more the returns to scale are overestimated (underestimated).

Often the most interesting results in production function estimation are, paradoxically, the residuals that are considered as the producers' unobservable productivity levels. In most of the cases of scenarios 1 and 4 the standard deviation of the productivity distribution is correctly estimated. Also the correlation between the estimated and the true firm-level productivity levels is very high, in some cases even equal to one. The exceptions are in fact cases 10 ($\beta_{L1} = 0.7$, $\beta_{K1} = 0.3$ and $\beta_{L2} = 0.71$, $\beta_{K2} = 0.3$), 13 ($\beta_{L1} = 0.7$, $\beta_{K1} = 0.3$ and $\beta_{L2} = 0.69$, $\beta_{K2} = 0.3$) and 16 ($\beta_{L1} = 0.71$, $\beta_{K1} = 0.3$ and $\beta_{L2} = 0.69$, $\beta_{K2} = 0.3$), where the product-level technologies are very similar but with small differences in the returns to scale. In these cases the correlations between the estimated and true firm-level productivity terms, $\exp(\widehat{\omega}_j)$ and $\exp(\omega_j)$, are 0.53, 0.53 and 0.31, respectively. The standard deviations of the productivity distributions are overestimated: they are estimated to be 0.19, 0.19 and 0.31, respectively, while the standard deviation of the true firm-level productivity aggregate is only 0.10.

Scenarios 2 and 3 differ from scenarios 1 and 4 in how the estimated and true firm-level productivity terms compare. First, with the exception of case 16, the estimated productivity distributions are narrower ($\exp(\widehat{\omega}_j)$ is between 0.08 and 0.16) than the true firm-level distributions ($\exp(\omega_j)$ is between 0.21 and 0.29). Second, the correlation between $\exp(\widehat{\omega}_j)$ and $\exp(\omega_j)$ ranges not higher than 0.19 to 0.41 (scenario 2) and 0.26 to 0.32 (scenario 3).

To sum up, estimations on the simulated datasets show that the biases in the estimated firm-level parameters are substantial even when the true product-level technologies are very similar. The directions and the magnitudes of the parameter biases are determined as intricate functions of the true product-level technologies and the product scopes of the firms in the industry. Also the residuals, which are often considered as the unobservable productivity levels, are affected: the more heterogeneous the product scopes of the firms, the lower the correlation between the estimated and true firm-level productivity levels.

5 Conclusion

In this study I consider the implications of misspecifying production functions as firm-level functions, when the true technologies are product-specific. This question is important because the standard practice in the empirical literature is to assume that firms' produce all of their output with a single technology, and that the technology parameters for all goods in the industry are the same. However, the empirical literature suggests that production technologies across goods are likely to vary, and firms' product assortments are heterogeneous within industries. I consider the specification biases by simulations, where the data generation process lacks the typical features that complicate empirical production function estimation.

I find that the firm-level parameter estimates are biased in virtually all cases considered. The directions of the biases vary depending on the true product-specific parameters. The magnitudes of the biases grow in the difference between the true technologies' returns to scale, and the share of single-product firms. When the firms' product scopes are heterogeneous, as in the manufacturing sector, for example, the estimated productivity levels have a relatively low correlation with the true productivity levels. The productivity differences may be overestimated when the firms' product scopes are equal but the technologies' returns to scale are different, and the productivity differences may be understimated when the firms' product scopes differ. These findings yield a clear recommendation for applied economists: one should carefully consider the level at which production functions are estimated.

The reason why production functions have been estimated at the firm- instead of the product-level is due to data constraints: while a large share of firms is multiproduct firms, and datasets report inputs only at the firm-level, product-level input allocation is largely unobservable. Hence either datasets that report input allocation by product titles, and/or methodological contributions to estimating the input allocations would be enable estimating product-level production functions. Whether production functions are in fact product-specific, and to what extent the technologies differ across products, is an empirical question yet to be answered.

References

Ackerberg, D., L. C. Benkard, S. Berry, and A. Pakes (2007): *Econometric Tools for Analyzing Market Outcomes*vol. 6 of *Handbook of Econometrics*, chap. 63. Elsevier, 1 edn.

Ackerberg, D., K. Caves, and G. Frazer (2006): "Structural identification of production functions," MPRA Paper 38349, University Library of Munich, Germany.

Bernard, A. B., J. B. Jensen, S. J. Redding, and P. K. Schott (2007): "Firms in International Trade," *Journal of Economic Perspectives*, 21(3), 105–130.

Bernard, A. B., S. J. Redding, and P. K. Schott (2009): "Products and Productivity," *Scandinavian Journal of Economics*, 111(4), 681–709.

——— (2010): "Multiple-Product Firms and Product Switching," *American Economic Review*, 100(1), 70–97.

——— (2011): "Multiproduct Firms and Trade Liberalization," *Quarterly Journal of Economics*, 126(3), 1271–1318.

Brown, R. S., D. W. Caves, and L. R. Christensen (1979): "Modelling the Structure of Cost and Production for Multiproduct Firms," *Southern Economic Journal*, 46(1), 256–273.

Caves, D. W., L. R. Christensen, and M. W. Tretheway (1989): "Flexible Cost Functions for Multiproduct Firms," *Review of Economics and Statistics*, 62(3), 477–481.

De Loecker, J., P. K. Goldberg, A. K. Khandelwal, and N. Pavcnik (2012): "Prices, Markups, and Trade Reform," NBER Working Papers 17925, National Bureau of Economic Research, Inc.

Eaton, B. C., and N. Schmitt (1994): "Flexible Manufacturing and Market Structure," *American Economic Review*, 84(4), 875–888.

Eckel, C., L. Iacovone, B. Javorcik, and J. P. Neary (2011): "Multi-Product Firms at Home and Away: Cost- versus Quality-based Competence," CEPR. Discussion Papers 8186, Centre for Economic Policy Research.

Felipe, J., and F. M. Fisher (2003): "Aggregation in Production Functions: What Applied Economists Should Know," *Metroeconomica*, 54, 208–262.

Fisher, F. M. (1969): "The Existence of Aggregate Production Functions," *Econometrica*, 37(4), 553–577.

Fisher, M. F. (1993): *Aggregation: Aggregate Production Functions and Related Topics. Collected Papers of Franklin M. Fisher.* Cambridge London: MIT Press.

Gandhi, A., S. Navarro, and D. Rivers (2013): "On the Identification of Production Functions: How Heterogeneous is Productivity?," .

Greene, W. H. (2002): *Econometric Analysis*. Prentice Hall, 5 edn.

Houthakker, H. S. (1955-1956): "The Pareto Distribution and the Cobb-Douglas Production Function in Activity Analysis," *Review of Economic Studies*, 23(1), 27–31.

Johnson, J. P., and D. P. Myatt (2003): "Multiproduct Quality Competition: Fighting Brands and Product Line Pruning," *American Economic Review*, 93(3), 748–774.

——— (2006): "Multiproduct Cournot Oligopoly," *RAND Journal of Economics*, 37(3), 583–601.

Ju, J. (2003): "Oligopolistic Competition, Technology Innovation, and Multiproduct Firms," *Review of International Economics*, 11(2), 346–359.

Klein, L. R. (1946a): "Macroeconomics and the Theory of Rational Behavior," *Econometrica*, 14(2), 93–108.

——— (1946b): "Remarks on the Theory of Aggregation," *Econometrica*, 14(4), 303–312.

Leontief, W. (1947a): "Introduction to a Theory of the Internal Structure of Functional Relationships," *Econometrica*, 15(4), 361–373.

——— (1947b): "A Note on the Interrelationship of Subsets of Independent Variables of a Continuous Function with Continuous First Derivatives," *Bulletin of the American Mathematical Society*, 53(4), 343–350.

Levinsohn, J., and A. Petrin (2003): "Estimating Production Functions Using Inputs to Control for Unobservables," *Review of Economic Studies*, 70(2), 317–341.

May, K. (1946): "The Aggregation Problem for a One-Industry Model," *Econometrica*, 14(4), 285–298.

Mayer, T., M. Melitz, and G. I. P. Ottaviano (2014): "Market Size, Competition, and the Product Mix of Exporters," *American Economic Review*, 104(2), 495–536.

Nataf, A. (1948): "Sur la possibilité de construction de certains macromodèles," *Econometrica*, 16(3), 232–244.

Olley, G. S., and A. Pakes (1996): "The Dynamics of Productivity in the Telecommunications Equipment Industry," *Econometrica*, 64(6), 1263–1297.

Panzar, J. (1989): *Technological determinants of firm and industry structure*vol. 1 of *Handbook of Industrial Organization*, chap. 1, pp. 3–59. Elsevier, 1 edn.

Pu, S. S. (1946): "A Note on Macroeconomics," *Econometrica*, 14(4), 299–302.

Roson, R. (2012): "Competition between Multiproduct Firms with Heterogeneous Costs," Working Papers $2012_14, Department of Economics, University of Venice "Ca' Foscari"$.

Valmari, N. (2014): "Estimating Production Functions of Multiproduct Firms," .

Wooldridge, J. (2009): "On estimating firm-level production functions using proxy variables to control for unobservables," *Economics Letters*, 104(3), 112–114.

6 Tables and Figures

Table 1: The true production function parameters

	Technology for good 1			Technology for good 2				
Case	β_{L1}	β_{K1}	$\sum\beta_1$	β_{L2}	β_{K2}	$\sum\beta_2$	$\sum\beta_1 = \sum\beta_2$	$\beta_{L1}-\beta_{L2}, \beta_{K1}-\beta_{K2}$
1	0.7	0.3	1	0.7	0.3	1	yes	0,0
2	0.7	0.3	1	0.5	0.5	1	yes	0.2,-0.2
3	0.7	0.3	1	0.3	0.7	1	yes	0.4,-0.4
4	0.71	0.3	1.01	0.71	0.3	1.01	yes	0,0
5	0.71	0.3	1.01	0.51	0.5	1.01	yes	0.2,-0.2
6	0.71	0.3	1.01	0.31	0.7	1.01	yes	0.4,-0.4
7	0.69	0.3	0.99	0.69	0.3	0.99	yes	0,0
8	0.69	0.3	0.99	0.49	0.5	0.99	yes	0.2,-0.2
9	0.69	0.3	0.99	0.29	0.7	0.99	yes	0.4,-0.4
10	0.7	0.3	1	0.71	0.3	1.01	no	0.01, 0
11	0.7	0.3	1	0.51	0.5	1.01	no	0.19,-0.2
12	0.7	0.3	1	0.31	0.7	1.01	no	0.39,-0.4
13	0.7	0.3	1	0.69	0.3	0.99	no	0.01,0
14	0.7	0.3	1	0.49	0.5	0.99	no	0.21,-0.2
15	0.7	0.3	1	0.29	0.7	0.99	no	0.41,-0.4
16	0.71	0.3	1.01	0.69	0.3	0.99	no	0.02,0
17	0.71	0.3	1.01	0.49	0.5	0.99	no	0.22,-0.2
18	0.71	0.3	1.01	0.29	0.7	0.99	no	0.42,-0.4

Table 2: Scenario 1; 1/2 of the firms produce good 1, and 1/2 of the firms produce good 2

	Estimated industry-specific technology					Technology for good 1				Technology for good 2				s.d. ω_j	$\sum \beta_1 = \sum \beta_2$
Case	$\widehat{\beta}_L$	$\widehat{\beta}_K$	$\sum \widehat{\beta}$	s.d. $\exp \widehat{\omega}_j$	$Corr\,(\widehat{\omega}_j, \omega_j)$	β_{L1}	β_{K1}	$\sum \beta_1$	$\frac{\sum Q_1}{\sum Q_1+Q_2}$	β_{L2}	β_{K2}	$\sum \beta_2$	$\frac{\sum Q_2}{\sum Q_1+Q_2}$		
1	0.70	0.30	1	0.10	1	0.7	0.3	1	0.5	0.7	0.3	1	0.5	0.10	yes
2	0.70	0.30	1	0.10	1	0.7	0.3	1	0.52	0.5	0.5	1	0.48	0.10	yes
3	0.50	0.50	1	0.10	0.99	0.7	0.3	1	0.5	0.3	0.7	1	0.5	0.10	yes
4	0.71	0.30	1.01	0.10	1	0.71	0.3	1.01	0.5	0.71	0.3	1.01	0.5	0.10	yes
5	0.71	0.30	1.01	0.10	1	0.71	0.3	1.01	0.52	0.51	0.5	1.01	0.48	0.10	yes
6	0.52	0.49	1.01	0.10	0.99	0.71	0.3	1.01	0.5	0.31	0.7	1.01	0.5	0.10	yes
7	0.69	0.30	0.99	0.10	1	0.69	0.3	0.99	0.5	0.69	0.3	0.99	0.5	0.10	yes
8	0.68	0.31	0.99	0.10	1	0.69	0.3	0.99	0.52	0.49	0.5	0.99	0.48	0.10	yes
9	0.48	0.51	0.99	0.10	0.99	0.69	0.3	0.99	0.5	0.29	0.7	0.99	0.5	0.10	yes
10	1.07	0.10	1.17	0.19	0.53	0.7	0.3	1	0.41	0.71	0.3	1.01	0.59	0.10	no
11	0.34	0.68	1.02	0.10	0.98	0.7	0.3	1	0.44	0.51	0.5	1.01	0.56	0.10	no
12	0.32	0.69	1.01	0.10	0.99	0.7	0.3	1	0.42	0.31	0.7	1.01	0.58	0.10	no
13	1.06	0.10	1.16	0.19	0.53	0.7	0.3	1	0.59	0.69	0.3	0.99	0.41	0.10	no
14	1.05	-0.06	0.98	0.10	0.96	0.7	0.3	1	0.61	0.49	0.5	0.99	0.39	0.10	no
15	0.68	0.32	1	0.10	0.99	0.7	0.3	1	0.58	0.29	0.7	0.99	0.42	0.10	no
16	2.06	-0.46	1.6	0.34	0.31	0.71	0.3	1.01	0.67	0.69	0.3	0.99	0.33	0.10	no
17	1.40	-0.41	0.99	0.11	0.89	0.71	0.3	1.01	0.69	0.49	0.5	0.99	0.31	0.10	no
18	0.86	0.14	1.00	0.10	0.97	0.71	0.3	1.01	0.67	0.29	0.7	0.99	0.33	0.10	no

Table 3: Scenario 2; 1/3 of the firms produce good 1, 1/3 of the firms produce good 2, and 1/3 of the firms produce both goods

	Estimated firm-level technology					Technology for good 1				Technology for good 2				s.d. ω_j	
Case	$\widehat{\beta}_L$	$\widehat{\beta}_K$	$\sum\widehat{\beta}$	s.d. $\exp\widehat{\omega}_j$	$Corr\,(\widehat{\omega}_j,\omega_j)$	β_{L1}	β_{K1}	$\sum\beta_1$	$\frac{\sum Q_1}{\sum Q_1+Q_2}$	β_{L2}	β_{K2}	$\sum\beta_2$	$\frac{\sum Q_2}{\sum Q_1+Q_2}$		$\sum\beta_1=\sum\beta_2$
1	0.70	0.30	1	0.09	0.21	0.7	0.3	1	0.50	0.7	0.3	1	0.50	0.24	yes
2	0.68	0.29	0.98	0.09	0.21	0.7	0.3	1	0.52	0.5	0.5	1	0.48	0.25	yes
3	0.45	0.45	0.9	0.09	0.19	0.7	0.3	1	0.50	0.3	0.7	1	0.50	0.24	yes
4	0.71	0.30	1.00	0.09	0.21	0.71	0.3	1.01	0.50	0.71	0.3	1.01	0.50	0.24	yes
5	0.69	0.29	0.98	0.09	0.21	0.71	0.3	1.01	0.52	0.51	0.5	1.01	0.48	0.25	yes
6	0.46	0.44	0.9	0.09	0.19	0.71	0.3	1.01	0.50	0.31	0.7	1.01	0.50	0.24	yes
7	0.70	0.31	1	0.09	0.21	0.69	0.3	0.99	0.50	0.69	0.3	0.99	0.50	0.24	yes
8	0.68	0.30	0.98	0.09	0.21	0.69	0.3	0.99	0.52	0.49	0.5	0.99	0.48	0.25	yes
9	0.44	0.46	0.9	0.09	0.19	0.69	0.3	0.99	0.50	0.29	0.7	0.99	0.50	0.24	yes
10	0.91	0.12	1.03	0.16	0.29	0.7	0.3	1	0.41	0.71	0.3	1.01	0.59	0.25	no
11	0.32	0.67	0.98	0.09	0.2	0.7	0.3	1	0.44	0.51	0.5	1.01	0.56	0.25	no
12	0.27	0.65	0.92	0.09	0.19	0.7	0.3	1	0.42	0.31	0.7	1.01	0.58	0.25	no
13	0.91	0.12	1.03	0.16	0.29	0.7	0.3	1	0.59	0.69	0.3	0.99	0.41	0.25	no
14	1.06	-0.05	1.01	0.1	0.22	0.7	0.3	1	0.61	0.49	0.5	0.99	0.39	0.26	no
15	0.63	0.28	0.91	0.09	0.19	0.7	0.3	1	0.58	0.29	0.7	0.99	0.42	0.25	no
16	1.51	-0.39	1.12	0.29	0.41	0.71	0.3	1.01	0.67	0.69	0.3	0.99	0.33	0.28	no
17	1.44	-0.37	1.07	0.11	0.22	0.71	0.3	1.01	0.69	0.49	0.5	0.99	0.31	0.29	no
18	0.84	0.11	0.95	0.10	0.19	0.71	0.3	1.01	0.67	0.29	0.7	0.99	0.33	0.28	no

Table 4: Scenario 3; 1/10 of the firms produce good 1, 1/10 of the firms produce good 2, and 8/10 of the firms produce both goods

	Estimated firm-level technology					Technology for good 1				Technology for good 2				s.d. ω_j	$\sum\beta_1=\sum\beta_2$
Case	$\widehat{\beta}_L$	$\widehat{\beta}_K$	$\sum\widehat{\beta}$	s.d. $\exp\widehat{\omega}_j$	$Corr\,(\widehat{\omega}_j,\omega_j)$	β_{L1}	β_{K1}	$\sum\beta_1$	$\frac{\sum Q_1}{\sum Q_1+Q_2}$	β_{L2}	β_{K2}	$\sum\beta_2$	$\frac{\sum Q_2}{\sum Q_1+Q_2}$		
1	0.70	0.30	1	0.08	0.29	0.7	0.3	1	0.5	0.7	0.3	1	0.5	0.21	yes
2	0.68	0.30	0.98	0.08	0.29	0.7	0.3	1	0.52	0.5	0.5	1	0.48	0.21	yes
3	0.45	0.45	0.90	0.08	0.27	0.7	0.3	1	0.50	0.3	0.7	1	0.50	0.21	yes
4	0.71	0.30	1	0.08	0.29	0.71	0.3	1.01	0.50	0.71	0.3	1.01	0.50	0.21	yes
5	0.69	0.29	0.98	0.08	0.29	0.71	0.3	1.01	0.52	0.51	0.5	1.01	0.48	0.21	yes
6	0.46	0.44	0.90	0.08	0.26	0.71	0.3	1.01	0.50	0.31	0.7	1.01	0.50	0.21	yes
7	0.70	0.31	1	0.08	0.30	0.69	0.3	0.99	0.50	0.69	0.3	0.99	0.50	0.21	yes
8	0.67	0.31	0.98	0.08	0.29	0.69	0.3	0.99	0.52	0.49	0.5	0.99	0.48	0.21	yes
9	0.44	0.46	0.90	0.08	0.27	0.69	0.3	0.99	0.50	0.29	0.7	0.99	0.50	0.21	yes
10	0.78	0.25	1.03	0.11	0.28	0.7	0.3	1	0.41	0.71	0.3	1.01	0.59	0.22	no
11	0.33	0.65	0.98	0.08	0.28	0.7	0.3	1	0.44	0.51	0.5	1.01	0.56	0.21	no
12	0.28	0.64	0.92	0.08	0.27	0.7	0.3	1	0.42	0.31	0.7	1.01	0.58	0.22	no
13	0.77	0.26	1.03	0.11	0.28	0.7	0.3	1	0.59	0.69	0.3	0.99	0.41	0.22	no
14	1.03	-0.02	1.01	0.08	0.30	0.7	0.3	1	0.61	0.49	0.5	0.99	0.39	0.22	no
15	0.63	0.28	0.91	0.08	0.27	0.7	0.3	1	0.58	0.29	0.7	0.99	0.42	0.22	no
16	0.99	0.11	1.10	0.17	0.30	0.71	0.3	1.01	0.67	0.69	0.3	0.99	0.33	0.23	no
17	1.39	-0.32	1.07	0.09	0.32	0.71	0.3	1.01	0.69	0.49	0.5	0.99	0.31	0.23	no
18	0.84	0.12	0.95	0.08	0.28	0.71	0.3	1.01	0.67	0.29	0.7	0.99	0.33	0.23	no

Table 5: Scenario 4; All firms produce both goods

	Estimated firm-level technology					Technology for good 1				Technology for good 2				s.d. ω_j	$\sum\beta_1 = \sum\beta_2$
Case	$\widehat{\beta}_L$	$\widehat{\beta}_K$	$\sum\widehat{\beta}$	s.d. $\exp\widehat{\omega}_j$	$Corr\,(\widehat{\omega}_j, \omega_j)$	β_{L1}	β_{K1}	$\sum\beta_1$	$\frac{\sum Q_1}{\sum Q_1+Q_2}$	β_{L2}	β_{K2}	$\sum\beta_2$	$\frac{\sum Q_2}{\sum Q_1+Q_2}$		
1	0.70	0.30	1.00	0.07	1	0.7	0.3	1	0.5	0.7	0.3	1	0.5	0.07	yes
2	0.62	0.38	1.00	0.07	1	0.7	0.3	1	0.53	0.5	0.5	1	0.47	0.07	yes
3	0.50	0.50	1.00	0.07	1	0.7	0.3	1	0.50	0.3	0.7	1	0.50	0.07	yes
4	0.71	0.30	1.01	0.07	1	0.71	0.3	1.01	0.50	0.71	0.3	1.01	0.50	0.07	yes
5	0.63	0.38	1.01	0.07	1	0.71	0.3	1.01	0.53	0.51	0.5	1.01	0.47	0.07	yes
6	0.51	0.50	1.01	0.07	1	0.71	0.3	1.01	0.50	0.31	0.7	1.01	0.50	0.07	yes
7	0.69	0.30	0.99	0.07	1	0.69	0.3	0.99	0.50	0.69	0.3	0.99	0.50	0.07	yes
8	0.61	0.38	0.99	0.07	1	0.69	0.3	0.99	0.53	0.49	0.5	0.99	0.47	0.07	yes
9	0.49	0.50	0.99	0.07	1	0.69	0.3	0.99	0.50	0.29	0.7	0.99	0.50	0.07	yes
10	0.71	0.30	1.01	0.07	0.99	0.7	0.3	1	0.41	0.71	0.3	1.01	0.59	0.07	no
11	0.54	0.47	1.01	0.07	0.99	0.7	0.3	1	0.44	0.51	0.5	1.01	0.56	0.07	no
12	0.39	0.61	1.01	0.07	0.99	0.7	0.3	1	0.41	0.31	0.7	1.01	0.59	0.07	no
13	0.70	0.30	1.00	0.07	0.99	0.7	0.3	1	0.59	0.69	0.3	0.99	0.41	0.07	no
14	0.70	0.29	1.00	0.07	0.99	0.7	0.3	1	0.62	0.49	0.5	0.99	0.38	0.07	no
15	0.61	0.39	1.00	0.07	1	0.7	0.3	1	0.59	0.29	0.7	0.99	0.41	0.07	no
16	0.72	0.30	1.02	0.08	0.96	0.71	0.3	1.01	0.68	0.69	0.3	0.99	0.32	0.08	no
17	0.80	0.21	1.01	0.08	0.96	0.71	0.3	1.01	0.70	0.49	0.5	0.99	0.30	0.08	no
18	0.72	0.28	1.01	0.08	0.98	0.71	0.3	1.01	0.67	0.29	0.7	0.99	0.33	0.08	no

Essay 3

Heterogeneous Productivity of Information Technology

Unpublished

Heterogeneous Productivity of Information Technology*

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Abstract

This study estimates the range of productivity gains achieved by information technology investments in the Finnish manufacturing sector. The contribution is to provide estimates of IT's productivity effects while accounting for some of the key characteristics of IT, i.e., that returns to IT depend on previous IT or complementary investments, come with lags, and, due to the aforementioned factors, are heterogeneous across firms and over time. I find that the productivity effects of IT range from negative to positive. For example, most firms obtain a negative productivity effect in the first year after the investment, which may be due to disruption in the production process caused by the implementation of the IT investment. Two years after the IT investment was made, most firms attain a positive productivity effect. In the third year after the investment, almost all firms gain a positive productivity effect. The estimation results suggest that the common practice of estimating a single output elasticity for an IT stock that is constructed as a linear function of the IT investments is unlikely to provide a truthful description of the productivity effects of IT.

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1 Introduction

This study estimates the range of productivity gains achieved by information technology investments (henceforth IT) in the Finnish manfacturing sector. The contribution is to provide estimates of IT's productivity effects that account for some of the key characteristics of IT, i.e., that returns to IT may depend on previous IT or other complementary investments, come with lags, and, due to the aforementioned factors, be heterogeneous across firms and over time. In other words, instead of estimating a single number to describe the output elasticity of IT, I estimate an entire distribution of productivity effects of IT. I take an agnostic view on how productive IT capital is formed and, in contrast to the rest of the literature where the IT capital stock is computed as a linear function of the past IT investments and some assumed depreciation rate, I endogenize the plant's productivity term to the past IT investments in a flexible way. I do this by estimating the model of endogenous productivity by Doraszelski and Jaumandreu (2013).

Implementation of more efficient technologies is one of the most important ways to enhance productivity growth. Information technology has been predicted to be the most important general purpose technology invented since electricity (Biagi, 2013). IT is expected to enhance firms' productivity by allowing for more efficient production processes, enable development of new goods and services, and thereby increase economic growth. Because IT expenditures and investments constitute a significant share of firms' spending, IT may be expected to have an important impact on firms' output. The estimated returns to IT vary across industries and countries (Draca, Sadun and Van Reenen, 2006). At the same time substantial productivity differences between firms even within narrowly defined national industries persist (Doms and Bartelsman, 2000; Syverson, 2011). A plausible explanation for the heterogeneous productivity effects of IT is that the productivity potential of IT realizes only when accompanied by complementary investments, and if firms' success in implementing them varies between industries, then also the returns to IT are likely to differ from one industry to another. If firms are heterogeneous with respect to IT complementarities also within industries, then the (lacking) complementarities may explain also some of the intra-industry productivity differences.

In this study I find that the productivity effects of IT are heterogeneous over firms and across time, ranging from negative to positive impacts. They depend on the productivity level previously attained as well as the IT investments made in other years. The productivity effects of an IT investment made in a given year also vary over the following years. For example, most firms obtain a negative productivity effect in the first year after the investment, which may be due to disruption in the production process caused by the implementation of the IT investment. Two years after the IT investment was made, most firms attain a positive productivity effect. In the third year after the investment, almost all firms gain a positive productivity effect. Variation in the productivity effects of IT over the years, and as a function of IT investments made in other years, suggest that at least some of the IT investments are complementary. In other words, the common practice of estimating a single output elasticity for an IT stock that is constructed as a linear function of the IT investments is unlikely to provide a truthful description of the productivity effects of IT.

The next section reviews the micro-level literature on IT and productivity. Section 3 presents the data used in this study. The model and the estimation strategies, slightly modified from Doraszelski and Jaumandreu (2013) for the application of this paper, are presented in sections 4 and 5. The results are presented and discussed in section 6. Section 7 concludes.

2 Literature

Numerous studies provide estimates on the productivity effects of IT investments. The earliest studies were carried out at the industry- or economy-level using the growth accounting approach (for a review, see Biagi, 2013). Since the 1990's the association between IT and productivity has been considered also at the firm-level (for an example, see Brynjolfsson and Hitt, 1996). Most of these micro-level studies examine the productivity effects of IT by estimating a production function where IT is one of the inputs, together with traditional non-IT capital and non-IT labor. Some of the earliest firm-level studies consider only investment in or use of hardware, i.e., computers, but in the majority of the studies the measure of IT includes also software investments and the labor expenses involved. A few studies treat the adoption of IT as a discrete choice. However the standard practice is to construct a firm-level IT capital stock as a linear function of the observable IT investments and expenditures, and some assumed depreciation rate.

Virtually all firm-level studies report a positive estimate of return to IT, even if the estimated productivity effects vary across industries and countries. A plausible explanation for the different estimates is related to IT being a so-called general purpose technology: IT becomes productive only when accompanied by complementary investments, and the more there are complements, the higher the returns to IT are likely to be. Firms' success in realizing the productivity potential of their IT investments therefore depends on the complementary investments they make. Moreover, if the complementary investments are carried out over a long time period, the productivity effects of IT may come with lags, or even be initially negative. It is also worthwhile noting that to obtain unbiased and consistent estimates of production functions, the relevant identification issues have to be accounted for. Firms' input choices are a function of the firms' productivity levels that are unobservable to the econometrician, and if these endogeneities are not controlled for, the parameter estimates are biased and inconsistent. Unfortunately, not correcting for these endogeneity biases is the rule rather than the exception in the literature on IT and productivity (for exceptions, see Barua and Lee, 1997; Aral, Brynjolfsson and Wu, 2006). Reviewing the whole micro-level literature is not feasible in this paper, but literature reviews on establishment-level studies on IT and productivity are provided by Brynjolfsson (1992), Brynjolfsson and Yang (1997), Stiroh (2005), Dedrick, Gurbaxani and Kraemer (2003), Draca, Sadun and Van Reenen (2006), and Biagi (2013).

Returns to IT investments depend on whether IT is a substitute or complement to other factors of production, and whether IT requires complementary investments to become productive. Dewan and Min (1997) and Hitt and Snir (1999) find evidence for IT being a substitute for labor. Their findings on the complementarity of IT and capital are mixed, however. Hitt and Snir find that firms' IT capital stock, which comprises computer capital and information system related labor expenses, is a substitute for ordinary capital. Hitt and Snir examine how organizational factors affect the substitutability of IT capital for ordinary factors of production. In what they call "modern" organizations, i.e., firms with decentralized organizational form, skilled staff, new non-IT capital, and small inventories, IT turns out to be a complement for non-IT capital. In "traditional" firms, i.e., firms without the aforementioned characteristics, IT and non-IT capital are substitutes. Ko and Osei-Bryson (2006) show that the returns to IT depend on the size of the investment as well as investments made in non-IT inputs such as ordinary capital and labor.

There are also other factors that are found to complement IT and enhance its productivity effects, but these factors are seldom considered in production function estimation. According to the current literature the most important complement to IT is organizational capital and management practices. For example, decentralized organizational form and decision making, team work, new human resource management practices, i.e., practices related to promotions, rewards, hiring and firing, and changes in the boundaries of the firm complement IT capital (Dewan and Min, 1997; Bresnahan, Brynjolfsson and Hitt, 2002; Brynjolfsson and Hitt, 2003; Dedrick, Gurbaxani and Kraemer, 2003; Zand, Van Beers and Van Leeuwen, 2011; Bloom, Sadun and Van Reenen, 2012). Another key complement is human capital, i.e., skilled labor enhances the productivity of IT investments (Bartel, Ichniowski and Shaw, 2005). One more complement is R&D, or innovation, in processes and products. The empirical findings on the complementarity of IT and R&D are mixed, however: Bresnahan, Brynjolfsson and Hitt (2002), van der Wiel, van Leeuwen, Hempell (2004), and Bartel, Ichniowski and Shaw (2005) find IT and product and/or process innovations to complement each other, while Hall, Lotti and Mairesse (2012) do not find complementarities between IT and R&D activities.

Van Leeuwen and Polder (2013) consider the fact that complementarities may exist also between different types of IT investments. They distinguish between three different kinds of E-business systems: Enterprise Resource Planning (ERP), Customer Relationship Management (CRM), and Supply Chain Management (SCM). They compare the productivity effects of adopting these systems separately and jointly. Van Leeuwen and Polder find that ERP and SCM are strong complements, and ERP and CRM are substitutes, while the results for the complementarity between CRM and SCM are contradictory.

Due to the importance of complementary investments, which may be carried out over a long time period, the productivity effects of IT may realize only years after the IT investment was made. Brynjolfsson and Hitt (2003) examine how the returns to IT vary over time. They estimate the effects of computerization on firms' output and productivity. Brynjolfsson and Hitt find computer investments to have normal returns in one year's time, and returns up to five times greater over five to seven years. This finding can be interpreted as evidence of large and time-consuming complementary investments taking place. Also the riskiness of IT investments has been acknowledged in the literature. Dewan, Shi and Gurbaxani (2007) evaluate the risks and returns of IT investments. They use two estimation frameworks: firm's production function and market value specifications. The IT risks are estimated at the industry-level. Dewan, Shi and Gurbaxani find that IT investments are riskier than non-capital investments. In addition, firms with higher IT risk also have higher marginal product of IT.

Also the environment in which firms operate may have an impact on how IT affects firms' productivity. Chang and Gurbaxani (2013) and Melville, Gurbaxani and Kraemer (2007) consider why firms in some sectors obtain higher returns to IT than firms in other sectors. They find that firms in more competitive markets make more IT investments than firms in non-competitive markets. However, the firms in more competitive environments get lower returns to IT because competition moderates them. In addition, Melville, Gurbaxani and Kraemer (2007) find limited evidence also for the marginal product of IT being higher in more dynamic industries, where the deviations of industry sales from a trend line are higher. Chang and Gurbaxani (2012a) find evidence for one more explanation for excessive returns to IT, along with complementarities and risk: spillovers. Chang and Gurbaxani study how IT-related spillovers through intraindustry transactions, in particular with the IT services industry, affect firms' productivity over a long-term horizon. They find that the spillovers have a significant impact, and that the magnitude and persistence of the impact are positively dependent on the IT intensity of the firm.

Many firms purchase internal information systems from service providers outside of the firm. Chang and Gurbaxani (2012b) evaluate the impact of IT outsourcing on firms' productivity. They show that IT outsourcing entails productivity effects, and that subsequently contracting out entails additional productivity gains. The productivity effects of outsourcing increase in the magnitude of oursourcing, but decrease in the firm's IT intensity.

3 Data

The data used in this study is from the Longitudinal Database on Plants in Finnish Manufacturing (LDPM), provided by Statistics Finland. The dataset comprises plant-level information on inputs and outputs, and various background information on the plants. I use the data from the years 1996 to 2008. During this time period, the sample includes all plants that belong to manufacturing firms with at least 20 employees. Each plant has a two-digit NACE (La nomenclature statistique des activités économiques dans la Communauté européenne) code that identifies the main industry of the plant.

The output variable is gross output, defined as the sum of sales revenue, deliveries to other plants of the firm, changes in inventories, production for own use, and other business revenue, deducting capital gains and acquisition of merchandise. There are four types of factors of production in the data that are used in estimating plants' production functions: materials, labor, physical capital, and information technology. Materials are measured in monetary value. The measure of labor input used is an equivalent for the number of full-time employees, and it is based on the number of hours worked. The labor costs observed are wages paid and the social costs. The capital stock is estimated using the perpetual inventory method[1] (PIM) as $K_t = K_{t-1}(1-\delta) + I_{t-1}$ where K_{t-1} is the capital stock in year $t-1$, δ is the depreciation rate assumed to be 0.1, and I_{t-1} are the investments made in year $t-1$. For plants that have become active before 1974 since when the data has been collected, the initial capital stock is assumed equal to the fire insurance valuation for capital. The measure of IT includes IT services purchased from outside of the firm. Examples of such IT services are consulting on hardware and software for automatic information processing, design and production of software, database management, repair and maintenance of computers, and information handling services. Gross output, materials and capital are measured in real value, fixing the price level of 2000 as the base level, and using the implicit price deflators of the national accounts.

I study plants[2] that manufacture electrical equipment (NACE code 27). Electrical equipment comprise electric motors, generators, transformers and electricity distribution and control apparatus (NACE 271), batteries and accumulators (272), wiring and wiring devices (273), electric lighting equipment (274), domestic appliances (275), and other electrical equipment (279). I observe 90 plants during years 1996 to 2008, with a total of 1185 plant observations.

IT investment intensity of the plants, measured as the ratio of IT investments to gross output, is described in Table 1. Most plants make rather small IT investments, worth less

[1] The physical capital stock computed in the LDPM is $K_t = K_{t-1}(1-\delta) + I_t$, i.e., investments are assumed to turn into productive capital already during the year of investment. I assume that investments become part of the physical capital stock not until in the following year, and modify the variable accordingly.

[2] Empirical studies on production functions differ in whether the estimations are done at the firm- or the plant-level. Production functions are specified at the firm-level typically when plant-level data is not available.

than 1% of gross output. However, the distribution of IT intensity is heavily skewed to the right, with some plants making IT investments worth more than 40% of their gross output. IT investments made in the previous five years are highly correlated with gross output, as shown in Table 2. The high correlation may be due to the fact that big plants can make bigger IT investments than smaller plants, or the correlation may be caused by productivity effects of IT.

4 Model

I follow Doraszelski and Jaumandreu's (2013) example in estimating firms' production functions and, in particular, firms' endogenous productivity. Doraszelski and Jaumandreu distinguish between two types of investments: investment in physical capital, and investment in knowledge through R&D expenditures. Physical capital is an observable input in the sense that it is computed given the past investments and a depreciation rate of capital, as $K_{jt} = \delta K_{jt-1} + I_{jt-1}$. Knowledge capital, on the other hand, is unobservable to the econometrician. Instead of assuming how R&D expenditures are transformed into productive knowledge capital, Doraszelski and Jaumandreu estimate how R&D expenditures affect firms' productivity and output.

I estimate the production function for plants instead of firms. The production function for plant j at time t is a Cobb-Douglas function with three observable inputs, materials M_{jt}, labor L_{jt}, and physical capital K_{jt}:

$$Q_{jt} = \exp(\beta_t) M_{jt}^{\beta_M} L_{jt}^{\beta_L} K_{jt}^{\beta_K} \exp(\omega_{jt}) \exp(e_{jt}) . \quad (1)$$

Annual trends in production that are common for all plants are captured by β_t. The plant's productivity level, denoted by ω_{jt}, is correlated over time. In addition, the plant's output is affected by a mean zero random shock e_{jt} that is uncorrelated over time and across plants. In contrast to the rest of the literature, Doraszelski and Jaumandreu endogenize the productivity process to R&D expenditures. They assume that the firm's productivity level ω_{jt} evolves as a controlled first-order Markov process, i.e., determined by the productivity level attained and the R&D expenditures made in the previous year.

My goal is to explain how the monetary value of the IT investment, C_{jt}, affects the plant's productivity over a longer period. For this reason, I make as few assumptions as possible on how IT investments affect the plant's output. First, IT investments may have lagged effects on productivity if, for example, the IT investments become productive only when complementary investments are made in, say, reorganization of processes or training of employees. Second, IT investments from different years may have interaction effects on productivity if the IT investments made in various years are part of a greater body of IT investments. Third, the effects of IT investments on the plant's productivity may depend on the productivity level attained. To allow for these nonlinearities, I assume that the plant's productivity is a function of the previous year's productivity, and the IT investments made in the previous five years (lower case letters denote logs):

$$\omega_{jt} = E\left[\omega_{jt} | \omega_{jt-1}, c_{jt-1}, ..., c_{jt-5}\right] + \xi_{jt}. \tag{2}$$

The expected level of productivity $E\left[\omega_{jt}|.\right]$ is approximated by a complete set of polynomials of degree one, denoted by function $g\left(\omega_{jt-1}, c_{jt-1}, ..., c_{jt-5}\right)$. A productivity shock that is mean independent of $\omega_{jt-1}, c_{jt-1}, ..., c_{jt-5}$ is captured by ξ_{jt}. The productivity shock includes all uncertainties that affect the plant's productivity, including uncertainties related to the IT investments, such as success in implementation.

The timing of production decisions, i.e., when $E\left[\omega_{jt}|.\right]$, ξ_{jt}, and e_{jt} are observed and when M_{jt}, L_{jt}, K_{jt}, and C_{jt} are set, is crucial for the identification of the production model, as discussed in section 5. At time $t-1$, the decision maker of the plant forms an expectation of the productivity level that would be attained in the next period, $E\left[\omega_{jt}|.\right]$, conditional on a given IT investment, C_{jt-1}. The decision maker sets C_{jt-1} and I_{jt-1}, and hence K_{jt}, to maximize the expected net present value of future cash flows. In the beginning of period t, the plant's decision maker observes the productivity shock ξ_{jt}, and sets M_{jt} and L_{jt} to maximize the profit of period t. After that the output shock e_{jt} takes place. In short, $C_{jt-1}, ..., C_{jt-5}$ and K_{jt-1} are uncorrelated with ξ_{jt}, while M_{jt} and L_{jt} are correlated with ξ_{jt}.

5 Identification and Estimation Strategy

The challenge in estimating the production function with endogenougs productivity is the same as when estimating any production function: inputs are set as a function of productivity ω_{jt} that is unobservable to the econometrician. As a consquence, if the endogeneity between ω_{jt} and the inputs is not controlled for, the parameter estimates are biased. Levinsohn and Petrin (2003) point out that because flexible input(s) such as M_{jt} and L_{jt} are set as a function of ω_{jt}, they contain information about the unobservable ω_{jt}. The demand function for M_{jt} or L_{jt} can be inverted to recover ω_{jt}, which can then be controlled for in the estimation equation. Doraszelski and Jaumandreu build on the insight of Levinsohn and Petrin. They note that for the estimated production function, the functional form of the demand functions for M_{jt} and L_{jt} are known, and hence a parametric expression for ω_{jt} can be derived.

The unobservable ω_{jt} is obtained from the solution to the plant's static profit maximization problem. More specifically, the profit-maximizing measure of (one of) the flexible input(s) is first solved, and then rewritten for ω_{jt}. I use L_{jt} as the control variable for ω_{jt} because the labor costs are observable, unlike material prices. The plant's decision maker's static profit maximization problem is:

$$\max_{M_{jt},L_{jt}} E\left[\Pi_{jt}\right] = Q_{jt} - P_{Mjt}M_{jt} - W_{jt}L_{jt}, \tag{3}$$

substituting in the production function:

$$\max_{M_{jt},L_{jt}} E\left[\Pi_{jt}\right] = \exp\left(\beta_t\right) M_{jt}^{\beta_M} L_{jt}^{\beta_L} K_{jt}^{\beta_K} \exp\left(\omega_{jt}\right) E\left[\exp\left(e_{jt}\right)\right] - P_{Mjt}M_{jt} - W_{jt}L_{jt}. \tag{4}$$

The first order condition for L_{jt} in static maximization is:

$$\frac{\partial E\left[\Pi_{jt}\right]}{\partial L_{jt}} = \beta_L \exp\left(\beta_t\right) M_{jt}^{\beta_M} L_{jt}^{\beta_L - 1} K_{jt}^{\beta_K} \exp\left(\omega_{jt}\right) E\left[\exp\left(e_{jt}\right)\right] - W_{jt} = 0 \tag{5}$$

which can be rewritten to obtain an expression for ω_{jt}, called h_{jt}, where the lower case letters denote logs:

$$h_{jt} = w_{jt} - \log\left(\beta_L\right) - \beta_t - \beta_M m_{jt} - \left(\beta_L - 1\right) l_{jt} - \beta_K k_{jt} - \log\left(E\left[\exp\left(e_{jt}\right)\right]\right). \tag{6}$$

The estimation equation is obtained by lagging h_{jt} to obtain h_{jt-1}, and substituting it together with $c_{jt-1}, ..., c_{jt-5}$ in the production function, written in logs:

$$q_{jt} = \beta_t + \beta_M m_{jt} + \beta_L l_{jt} + \beta_K k_{jt} + g\left(h_{jt-1}, c_{jt-1}, ..., c_{jt-5}\right) + \xi_{jt} + e_{jt}. \tag{7}$$

Finally, the residual of the estimation equation is:

$$\xi_{jt} + e_{jt} = q_{jt} - \beta_t - \beta_M m_{jt} - \beta_L l_{jt} - \beta_K k_{jt} - g\left(h_{jt-1}, ..., h_{jt-5}, c_{jt-1}, ..., c_{jt-5}\right). \tag{8}$$

The parameters to be estimated are β_M, β_L, β_K, time dummies β_t, and 22 coefficients in the polynomial approximation $g(.)$.[3] The moment conditions that identify parameters β_M, β_L and β_K are based on the timing assumptions of the model, and are also familiar from previous production function estimation strategies, such as Olley and Pakes (1996), Levinsohn and Petrin (2003), Ackerberg, Caves and Frazer (2006), and Wooldridge (2009).

The time effects β_t are identified with year dummies. The flexible inputs m_{jt} and l_{jt} are endogenous to $\xi_{jt} + e_{jt}$, and hence m_{jt} and l_{jt} are not valid instruments. However lagged material input m_{jt-1} is a valid instrument for m_{jt} because the input choices are correlated over time, due to, for example, the correlation between ω_{jt} and ω_{jt-1}, while m_{jt-1} is uncorrelated with $\xi_{jt}+e_{jt}$. Similarly, lagged labor input l_{jt-1} is a valid instrument for l_{jt}. In addition, labor cost w_{jt} is a valid instrument for l_{jt} because l_{jt} is correlated with w_{jt}, but w_{jt} is uncorrelated with $\xi_{jt} + e_{jt}$. The physical capital stock k_{jt} is predetermined in period $t-1$, and is therefore exogenous to $\xi_{jt} + e_{jt}$. The parameters in $g(.)$ that govern the evolution of productivity are also exogenous to $\xi_{jt} + e_{jt}$.

Note that h_{jt-1} is solved as a function of w_{jt-1}, m_{jt-1}, l_{jt-1} and k_{jt-1}. Recall also that $g(.)$ is a function of $\omega_{jt-1}, c_{jt-1}, ..., c_{jt-5}$, approximated by first order polynomials of $\omega_{jt-1}, c_{jt-1}, ..., c_{jt-5}$. This implies that w_{jt-1}, m_{jt-1}, l_{jt-1}, k_{jt-1} and c_{jt-i}, where $i = 1, ..., 5$, and their interactions, are all valid instruments.

The production function is estimated by two-step GMM. The GMM objective function is

$$\min_{\theta} \left[\frac{1}{N}\sum_j A\left(z_j\right)\nu_j\left(\theta\right)\right]' W_N \left[\frac{1}{N}\sum_j A\left(z_j\right)\nu_j\left(\theta\right)\right] \tag{9}$$

[3]The 22 coefficients are for h_{jt-1}, c_{jt-1}, ... c_{jt-5} and their interactions, and a constant.

where $A(z_j)$ is a matrix of instruments of size $L \times T_j$, and $\nu_j(\theta)$ is the vector of residuals $\xi_{jt} + e_{jt}$ of size $T_j \times 1$. N is the number of plants, L is the number of instruments, and T_j is the number of observations of plant j. To reduce the complexity of the GMM estimation routine I "concentrate out"[4] the parameters that enter the GMM objective function linearly. These "linear" parameters are β_t and all the coefficients in $g(.)$. $A(z_j)$ comprises the instruments for identifying β_M, β_L, β_K. As discussed above, they are w_{jt-1}, m_{jt-1}, l_{jt-1}, k_{jt-1} and c_{jt-i}, where $i = 1, ..., 5$, and their interactions, and the second and third powers of these variables and interactions, as well as w_{jt} and k_{jt}. In the first step I use weight matrix

$$W_N = \left(\frac{1}{N} \sum_j A(z_j) \nu A(z_j)' \right)^{-1}, \tag{10}$$

and in the second step I use

$$W_N = \left(\frac{1}{N} \sum_j A(z_j) \nu_j \left(\widehat{\theta}\right) \nu_j \left(\widehat{\theta}\right)' A(z_j)' \right)^{-1}. \tag{11}$$

The production function estimates are obtained by minimizing the GMM objective function by the Gauss-Newton algorithm. To find the global minimum of the objective function, I use a large set of alternative starting values.

6 Results

The production function estimates are presented in Table 3. The parameter estimates for the traditional inputs, materials, labor, and capital, sum up to 1.09, which implies that the production technology has increasing returns to scale. However, the estimates of β_M, β_L and β_K are not statistically significant. Hansen's J-test does not reject the null hypothesis of valid overidentification restrictions for the non-linear parameters β_M, β_L and β_K estimated by GMM (Prob[Chi-sq.(23)>J] is 1.00).

The polynomial approximation of $E[\omega_{jt} | \omega_{jt-1}, c_{jt-1}, ..., c_{jt-5}]$ describes how productivity evolves as a function of the productivity level attained in the previous year, and the IT investments made in the five previous years. Three of the parameter estimates,

[4] This means estimating the linear parameters by OLS for any given set of non-linear parameters $\widehat{\beta}_M, \widehat{\beta}_L$ and $\widehat{\beta}_K$.

for ω_{t-1}, IT_{t-1} and $\omega_{t-1} * IT_{t-1}$, are significant at the 99% level, and six parameters, for IT_{t-2}, IT_{t-5}, $\omega_{t-1} * IT_{t-2}$, $\omega_{t-1} * IT_{t-5}$, $IT_{t-1} * IT_{t-5}$ and a constant, are significant at the 95% level. The estimated productivity distribution, including the productivity residuals $\xi_{jt} + e_{jt}$, is shown in Figure 4. The percentiles of the productivity distribution are summarized in Table 5. For any given set of inputs, the median firm in the productivity distribution produces 70% more than the firm at the 10th percentile, and 27% more than the firm at the 25th percentile of the productivity distribution. On the other hand, the firm at the 75th percentile is 31% more productive than the median firm, and the firm at the 90th percentile is 52% more productive than the median firm. In other words, the firm at the 90th percentile produces about 2.5 more than the 10th percentile firm with any given set of inputs. These productivity differences are remarkably large but differences of similar magnitude have been reported also for other industries and countries.[5]

Persistence in productivity, that is, the proportion of productivity attained that is transferred to the next year, depends on the productivity level and the IT investments made in the previous five years. The levels of persistence therefore vary across firms and years. The level of persistence is computed as $\frac{\partial g(\omega_{jt-1}, c_{jt-1}, ..., c_{jt-5})}{\partial \omega_{jt-1}}$, and its quantiles are reported in Table 6. The median firm in the persistence distribution retains 95% of the productivity level gained in the previous year. Even the 10th percentile firm retains 88% of its productivity level, and the 90th percentile firm preserves as much as 99% of its productivity. The levels of persistence are very high throughout the persistence distribution. The parameter estimates for $g(\omega_{jt-1}, c_{jt-1}, ..., c_{jt-5})$ imply that IT investments made in years $t-1$, $t-3$ and $t-4$ have a negative impact on productivity persistence (i.e., the estimated coefficients on $\omega_{t-1} * IT_{t-1}$, $\omega_{t-1} * IT_{t-3}$ and $\omega_{t-1} * IT_{t-4}$ are negative), as reported in Table 3, perhaps because their implementation disrupts the production process. The IT investments made in years $t-2$ and $t-5$, on the other hand, have a positive effect on productivity persistence (i.e., the estimated coefficients on $\omega_{t-1} * IT_{t-2}$ and $\omega_{t-1} * IT_{t-5}$ are positive).

The productivity effects of IT investments made in the previous years, $t-1$ to $t-5$, are also determined as a function of the productivity level gained and the IT investments made.

[5]Syverson (2004) reports that in four-digit SIC industries of the US manufacturing sector, the plant at the 90th percentile of the productivity distribution is, on average, almost twice as productive as the plant at the 10th percentile. Hsieh and Klenow (2009) report that in China and India, the plant at the 90th percentile is more than five times as productive as the plant at the 10th percentile of the distribution.

They are computed as $\frac{\partial g(h_{jt-1},...,h_{jt-5},c_{jt-1},...,c_{jt-5})}{\partial c_{jt-1}}$, $\frac{\partial g(h_{jt-1},...,h_{jt-5},c_{jt-1},...,c_{jt-5})}{\partial c_{jt-2}}$, $\frac{\partial g(h_{jt-1},...,h_{jt-5},c_{jt-1},...,c_{jt-5})}{\partial c_{jt-3}}$, $\frac{\partial g(h_{jt-1},...,h_{jt-5},c_{jt-1},...,c_{jt-5})}{\partial c_{jt-4}}$, and $\frac{\partial g(h_{jt-1},...,h_{jt-5},c_{jt-1},...,c_{jt-5})}{\partial c_{jt-5}}$. The quantiles of the productivity effects are reported in Table 6. The productivity effects of the previous year's IT investments, IT_{t-1}, are negative for most of the firms, but a minority of firms obtain a positive productivity effect already in the year after the investment. The productivity effect is an outcome of the positive linear effect of IT_{jt-1}, the negative complementarity between ω_{jt-1} and IT_{jt-1}, and IT_{jt-1} and IT_{jt-5}, and the positive complementarity between IT_{jt-1} and IT_{jt-2}, IT_{jt-1} and IT_{jt-3}, and IT_{jt-1} and IT_{jt-4}, as reported in Table 3.

The productivity effects of IT_{t-2} are on average positive, most of the firms obtaining a positive effect, but for some of the firms IT_{t-2} has a negative impact on productivity. On the one hand, ω_{jt-1}, IT_{jt-1} and IT_{jt-5} are positive complements to IT_{jt-2}, and on the other hand, IT_{jt-3} and IT_{jt-4} are negative complements to IT_{jt-2}. Also the linear effect of IT_{jt-2} is negative. Only for IT_{jt-3} the productivity effects are positive for the vast majority of the firms. The impacts of IT_{t-4} and IT_{t-5} are negative or positive, again depending on the productivity level and the other IT investments made.

The production function estimates imply that IT investments made in other years and also the productivity level previously attained affect the productivity effect of an IT investment. In the estimations $E\left[\omega_{jt}|\omega_{jt-1}, c_{jt-1}, ..., c_{jt-5}\right]$ is approximated by a first order polynomial for simplicity, and hence the signs of the complementarities are determined simply by the coeffiecients on $IT_{t-1} * IT_{t-2}$, $IT_{t-1} * IT_{t-3}$, and so forth. However, if the productivity process was approximated by a higher order polynomial, the signs of the complementarities would also depend on the productivity level previously gained and the IT investments made.[6]

To sum up, the estimation results imply that the productivity effects of IT are heterogeneous over firms and across time, ranging from negative to positive. They depend on the productivity level previously attained as well as the IT investments made in other years. The productivity effects of an IT investment made in a given year also vary over the following years. For example, most firms obtain a negative productivity effect in the first year after the investment, which may be due to disruption in the production process

[6] The signs of the complementarities would be determined by $\frac{\partial^2 g(\omega_{jt-1},c_{jt-1},...,c_{jt-5})}{\partial c_{jt-1}\partial c_{jt-2}}$, $\frac{\partial^2 g(\omega_{jt-1},c_{jt-1},...,c_{jt-5})}{\partial c_{jt-1}\partial c_{jt-3}}$, and so forth.

caused by the implementation of the IT investment. Two years after the IT investment was made, most firms attain a positive productivity effect. In the third year after the investment, almost all firms gain a positive productivity effect. Variation in the productivity effects of IT over the years, and as a function of IT investments made in other years, suggest that at least some of the IT investments are complementary. In other words, the common practice of estimating a single output elasticity for an IT stock that is constructed as a linear function of the IT investments is unlikely to provide a truthful description of the productivity effects of IT.

6.1 Discussion

The measure of output used in the estimations is sales revenue. Without data on output quantities or prices it is not possible to distinguish between productive efficiency and quality effects of IT. Hence the estimated returns to IT include both efficiency and quality effects of IT on plants' sales revenue.

There is one data limitation that should be kept in mind when interpreting the estimation results. The IT variable includes only those expenses that result from purchasing IT services from outside of the firm. In other words, spending on hardware, such as computers, and developing and operating IT systems in-house, by employees of the firm, are not included in the IT measure used. If hardware investments and in-house IT work consistitute a substantial share of plants' IT expenditures, the production function estimates are biased. The signs of these measurement error biases depend on how the measurement error, i.e., hardware investments and in-house IT work, are correlated with the unobservable, i.e., productivity. The correlation may be positive or negative. Also organizational capital and management practices are unobservable, while they potentially are key determinants of the productivity effects of IT. If IT and organizational capital are positively correlated, the returns to IT are overestimated.

7 Conclusion

The literature on information technology and productivity shows that IT improves firms' productivity, and that complementary investments such as management practices, skilled labor and innovation activities are key determinants of the productivity effects of IT. This study contributes to the literature by taking into account that the productivity effects of IT may depend also on the IT investments made in other years, and that the productivity effect of an IT investment made in a given year may come with a lag and also vary over the following years. Allowing for the aforementioned factors I find that the productivity effects of IT are heterogeneous over firms and years, ranging from negative to positive impacts. These findings suggest that the common practice of estimating a single output elasticity for an IT stock that is constructed as a linear function of the IT investments is unlikely to provide a truthful description of the productivity effects of IT.

The relationship between IT investments and firms' productivity has been widely studied among economists and professionals of other fields. Yet the question of how IT affects firms' productivity is not fully answered, and several open questions remain. First, to what extent IT enhances productive efficiency, and how much IT contributes to improved product quality and product innovation? Second, how does the environment in which the firm operates affect the productivity effects the firm gains for its IT investments? Third, is it only due to complementary investments that some firms succeed in implementing IT while some don't? New, more detailed datasets will hopefully aid answering these questions.

References

ACKERBERG, D., K. CAVES, AND G. FRAZER (2006): "Structural identification of production functions," MPRA Paper 38349, University Library of Munich, Germany.

BARTEL, A. P., C. ICHNIOWSKI, AND K. L. SHAW (2005): "How Does Information Technology Really Affect Productivity? Plant-Level Comparisons of Product Innovation, Process Improvement and Worker Skills," NBER Working Papers 11773, National Bureau of Economic Research, Inc.

BARUA, A., AND B. LEE (1997): "The Information Technology Productivity Paradox Revisited: A Theoretical and Empirical Investigation in the Manufacturing Sector," *International Journal of Flexible Manufacturing Systems*, 9(2), 145–166.

BIAGI, F. (2013): "ICT and Productivity: A Review of the Literature," Digital Economy Working Paper 2013/19, Institute for Prospective Technological Studies, European Commission.

BRESNAHAN, T. F., E. BRYNJOLFSSON, AND L. M. HITT (2002): "Information Technology, Workplace Organization, And The Demand For Skilled Labor: Firm-Level Evidence," *Quarterly Journal of Economics*, 117(1), 339–376.

BRYNJOLFSSON, E. (1992): "The Productivity Paradox of Information Technology: Review and Assessment," Working Paper Series 130, MIT Center for Coordination Science.

BRYNJOLFSSON, E., AND L. HITT (1996): "Paradox Lost? Firm-Level Evidence on the Returns to Information Systems Spending," *Management Science*, 42(4), 541–558.

BRYNJOLFSSON, E., AND L. M. HITT (2003): "Computing Productivity: Firm-Level Evidence," *Review of Economics and Statistics*, 85(4), 793–808.

BRYNJOLFSSON, E., AND S. YANG (1997): "Information Technology and Productivity: A Review of the Literature," Working Paper Series 202, MIT Center for Coordination Science.

CHANG, Y., AND V. GURBAXANI (2013): "An Empirical Analysis of Technical Efficiency: The Role of IT Intensity and Competition," *Information Systems Research*, 24(3), 561–578.

CHANG, Y. B., AND V. GURBAXANI (2012a): "The Impact of IT-Related Spillovers on Long-Run Productivity: An Empirical Analysis," *Information Systems Research*, 23(3), 868–886.

——— (2012b): "Information Technology Outsourcing, Knowledge Transfer, and Firm Productivity: An Empirical Analysis," *Management Information Systems Quarterly*, 36(4), 1043–1063.

DEDRICK, J., V. GURBAXANI, AND K. L. KRAEMER (2003): "Information technology and economic performance: A critical review of the empirical evidence," *ACM Computing Surveys*, 35(1), 1–28.

Dewan, S., and C. ki Min (1997): "The Substitution of Information Technology for Other Factors of Production: A Firm Level Analysis," *Management Science*, 43(12), 1660–1675.

Dewan, S., C. Shi, and V. Gurbaxani (2007): "Investigating the Risk-Return Relationship of Information Technology Investment: Firm-Level Empirical Analysis," *Management Science*, 53(12), 1829–1842.

Doms, M., and E. J. Bartelsman (2000): "Understanding Productivity: Lessons from Longitudinal Microdata," *Journal of Economic Literature*, 38(3), 569–594.

Doraszelski, U., and J. Jaumandreu (2013): "RD and productivity: Estimating endogenous productivity," *Review of Economic Studies*, (80), 1338–1383.

Draca, M., R. Sadun, and J. V. Reenen (2006): "Productivity and ICT: A Review of the Evidence," CEP Discussion Paper 749, Centre for Economic Performance.

Hall, Bronwyn H., F. L., and J. Mairesse (2012): "Evidence on the Impact of RD and ICT Investment on Innovation and Productivity in Italian Firms," NBER Working Papers 18053, National Bureau of Economic Research, Inc.

Hitt, L. M., and E. M. Snir (1999): "The Role of Information Technology in Modern Production: Complement or Substitute to Other Inputs?," Discussion paper, University of Pennsylvania, Wharton School.

Hsieh, C.-T., and P. J. Klenow (2009): "Misallocation and Manufacturing TFP in China and India," *Quarterly Journal of Economics*, 124(4), 1403–1448.

Ko, M., and K.-M. Osei-Bryson (2006): "Analyzing the impact of information technology investments using regression and data mining techniques," *Journal of Enterprise Information Management*, 19(4), 403–417.

Levinsohn, J., and A. Petrin (2003): "Estimating Production Functions Using Inputs to Control for Unobservables," *Review of Economic Studies*, 70(2), 317–341.

Melville, N., V. Gurbaxani, and K. Kraemer (2007): "The productivity impact of information technology across competitive regimes: The role of industry concentration and dynamism," *Decision Support Systems*, 43(1), 229–242.

Olley, G. S., and A. Pakes (1996): "The Dynamics of Productivity in the Telecommunications Equipment Industry," *Econometrica*, 64(6), 1263–1297.

Stiroh, K. J. (2005): "Reasessing the Impact of IT in the Production Function: A Meta-Analysis and Sensitivity Tests," *Annales d'Economie et de Statistique*, (79-80), 529–561.

Syverson, C. (2004): "Product Substitutability and Productivity Dispersion," *Review of Economics and Statistics*, 86(2), 534–550.

——— (2011): "What Determines Productivity?," *Journal of Economic Literature*, 49(2), 326–365.

van der Wierl, H., G. van Leeuwen, and T. Hempell (2004): "ICT, Innovation and Business Performance in Services: Evidence for Germany and the Netherlands," ZEW Discussion Papers 04-06, ZEW - Zentrum für Europäische Wirtschaftsforschung / Center for European Economic Research.

WOOLDRIDGE, J. (2009): "On estimating firm-level production functions using proxy variables to control for unobservables," *Economics Letters*, 104(3), 112–114.

ZAND, F., C. V. BEERS, AND G. V. LEEUWEN (2011): "Information technology, organizational change and firm productivity: A panel study of complementarity effects and clustering patterns in Manufacturing and Services," MPRA Paper 46469, Delft University of Technology.

Table 1: IT investment intensity

Quintile	0.10	0.25	0.50	0.75	0.90	Mean	Std dev
	0.000	0.001	0.003	0.006	0.011	0.006	0.021
Number of obs.	1185						

Table 2: Correlation of gross output at time t and IT investments made in years $t-5$ to $t-1$

	IT_{t-1}	IT_{t-2}	IT_{t-3}	IT_{t-4}	IT_{t-5}
Q_t	0.86	0.80	0.69	0.63	0.66
Number of obs.	425				

Table 3: Parameter estimates (continues on the next page)

	Parameter estimate (standard error)		Parameter estimate (standard error)
Materials	0.29 (2.18)	$\omega_{t-1} * IT_{t-1}$	-0.0067^{**} (0.0025)
Labor	0.24 (0.94)	$\omega_{t-1} * IT_{t-2}$	0.0044^{*} (0.0025)
Capital	0.56 (1.14)	$\omega_{t-1} * IT_{t-3}$	$-6.5431e-04$ (0.0026)
ω_{t-1}	0.9543^{**} (0.0260)	$IT_{t-1} * IT_{t-3}$	$2.9313e-04$ (0.0002)
IT_{t-1}	0.0518^{**} (0.0183)	$IT_{t-1} * IT_{t-4}$	$1.4461e-04$ (0.0002)
IT_{t-2}	-0.0355^{*} (0.0182)	$IT_{t-1} * IT_{t-5}$	$-4.9400e-04^{*}$ (0.0002)
IT_{t-3}	0.0039 (0.0188)	$IT_{t-2} * IT_{t-3}$	$-2.7552e-04$ (0.0002)
IT_{t-4}	0.0082 (0.0200)	$IT_{t-2} * IT_{t-4}$	$-1.9539e-04$ (0.0002)
IT_{t-5}	-0.0367^{*} (0.0197)	$IT_{t-2} * IT_{t-5}$	$1.3688e-04$ (0.0002)
$\omega_{t-1} * IT_{t-4}$	-0.0017 (0.0028)	$IT_{t-3} * IT_{t-4}$	$-9.2881e-05$ (0.0002)
$\omega_{t-1} * IT_{t-5}$	0.0049^{*} (0.0028)	$IT_{t-3} * IT_{t-5}$	$3.2669e-04$ (0.0002)
$IT_{t-1} * IT_{t-2}$	$2.5356e-05$ (0.0002)	$IT_{t-4} * IT_{t-5}$	$-1.6190e-04$ (0.0002)

* significant at 95% level; ** significant at 99% level

Table 3 continued

	Parameter estimate (standard error)		Parameter estimate (standard error)
constant	0.3094* (0.1808)	year '05	0.0546 (0.00364)
year '02	−0.0094 (0.0361)	year '06	0.0279 (0.0361)
year '03	−0.0349 (0.0363)	year '07	0.0167 (0.0353)
year '04	0.0067 (0.0359)	year '08	0.0490 (0.0350)

* significant at 95% level; ** significant at 99% level

Prob[Chi-sq.(23)>J]	1.0000
Number of obs.	425

Figure 4: Estimated productivity distribution, controlling for year effects

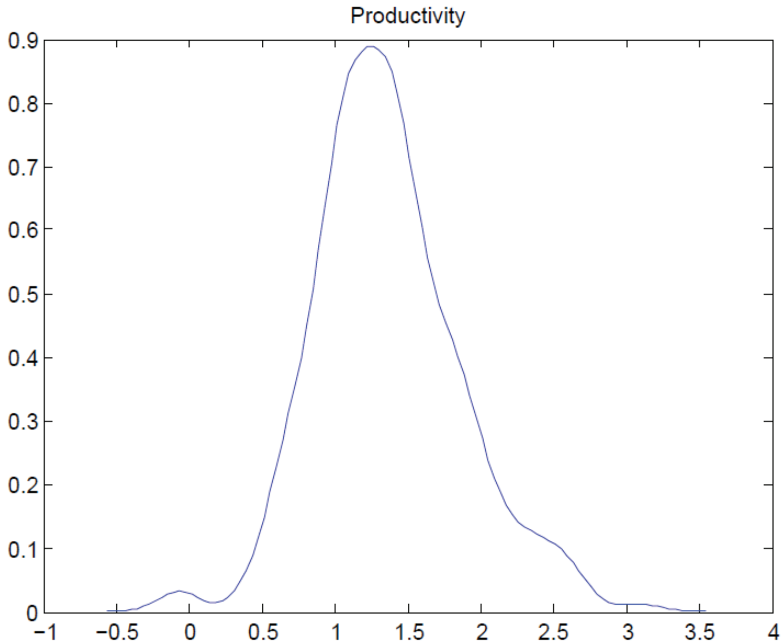

Table 5: Quintiles of estimated productivity

Quintile	0.10	0.25	0.50	0.75	0.90
	0.78	1.03	1.32	1.63	2.00
Number of obs.	425				

Table 6: Quintiles of estimated persistence in productivity and productivity effects of IT investments made in years $t-5$ to $t-1$

Quintile	0.10	0.25	0.50	0.75	0.90
$\frac{\partial g(\omega_{jt-1},c_{jt-1},\ldots,c_{jt-5})}{\partial \omega_{jt-1}}$	0.8789	0.9453	0.9508	0.9556	0.9850
$\frac{\partial g(\omega_{jt-1},c_{jt-1},\ldots,c_{jt-5})}{\partial c_{jt-1}}$	−0.0113	−0.0038	−0.0019	−0.0003	0.0006
$\frac{\partial g(\omega_{jt-1},c_{jt-1},\ldots,c_{jt-5})}{\partial c_{jt-2}}$	−0.0040	−0.0003	0.0009	0.0019	0.0031
$\frac{\partial g(\omega_{jt-1},c_{jt-1},\ldots,c_{jt-5})}{\partial c_{jt-3}}$	0.0000	0.0002	0.0008	0.0023	0.0172
$\frac{\partial g(\omega_{jt-1},c_{jt-1},\ldots,c_{jt-5})}{\partial c_{jt-4}}$	−0.0061	−0.0002	0.0006	0.0013	0.0020
$\frac{\partial g(\omega_{jt-1},c_{jt-1},\ldots,c_{jt-5})}{\partial c_{jt-5}}$	−0.0047	−0.0007	0.0005	0.0016	0.0045
Number of obs.	425				

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